

Information Capacity of Energy Harvesting Sensor Nodes

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Abstract—Sensor nodes with energy harvesting sources are gaining popularity due to their ability to improve the network life time and are becoming a preferred choice supporting ‘green communication’. We study such a sensor node with an energy harvesting source and compare various architectures by which the harvested energy is used. We find its Shannon capacity when it is transmitting its observations over an AWGN channel and show that the capacity achieving energy management policies are related to the throughput optimal policies. We also obtain the capacity when energy conserving sleep-wake modes are supported and an achievable rate for the system with inefficiencies in energy storage.

Keywords: Optimal energy management policies, energy harvesting, sensor networks, energy buffer, network life time.

I. INTRODUCTION

In wireless sensor networks, energy harvesting techniques are gaining popularity as means of increasing the battery life time and thereby the network life time ([1]). An energy harvester harnesses energy from its environment or other energy sources (e.g., body heat) and converts them to electrical energy. Harvesting solar energy through photovoltaic effect seems to have emerged as a technology of choice for many sensor nodes ([2]). Unlike for a battery operated sensor node, now there is potentially an *infinite* amount of energy available to the node. However, the source of energy and the energy harvesting device may be such that the energy cannot be generated at all times (e.g., a solar cell). Furthermore the rate of generation of energy can be limited. Thus one may want to match the energy generation profile of the harvesting source with the energy consumption profile of the sensor node. If the energy can be *stored* in the sensor node then this matching can be considerably simplified. But the energy storage device may have limited capacity. The energy consumption policy should be designed in such a way that the node can perform satisfactorily for a long time, i.e., at least energy starvation, should not be the reason for the node to die. In [1] such an energy/power management scheme is called *energy neutral operation*.

Energy harvesting can be often divided into two major architectures ([3]). In *Harvest-use*(HU), the harvesting system directly powers the sensor node and when sufficient energy

is not available the node is disabled. In *Harvest-Store-Use* (HSU) there is a storage device that stores the harvested energy and also powers the sensor node.

Various throughput and delay optimal energy management policies for energy harvesting sensor nodes are provided in [4]. The energy management policies in [4] are extended in various directions in [5] and [6]. For example, [5] also provides some efficient MAC policies for energy harvesting nodes. In [6] optimal sleep-wake policies are obtained for such nodes.

None of the above studies considers information-theoretic capacity of energy harvesting sensor nodes. We consider this problem in this paper. This problem is also addressed independently in [7]. However our model is more general and uses different proof techniques. Furthermore we compute the capacity when the energy is consumed in other activities at the node (e.g., processing, sensing etc) than transmission. This issue in the context of usual AWGN channel is addressed in [8]. Finally we provide the achievable rates when there are storage inefficiencies. We show that the throughput optimal policies provided in [4] are related to the capacity achieving policies provided here.

Related literature for conserving energy but without energy harvester is [9]-[10]. In [9] an explicit model for power consumption at an idealized decoder is studied. Optimal constellation size for uncoded transmission subject to peak power constraint is given in [11]. Reference [10] characterizes the capacity when the transmitter and the receiver probe the state of the channel. The probing action is cost constrained.

The paper is organized as follows. Section II describes the system model. Section III provides the capacity for a single node under idealistic assumptions. Section IV takes into account the energy spent on sensing, computation etc. and proposes capacity achieving sleep-wake schemes. Section V obtains efficient policies with inefficiencies in the energy storage system. Section VI concludes the paper.

II. MODEL AND NOTATION

In this section we present our model for a single energy harvesting sensor node. We consider a sensor node (Fig. 1) which is sensing and generating data to be transmitted to a central node via a discrete time AWGN Channel. We assume that transmission consumes most of the energy in a sensor node and ignore other causes of energy consumption (this

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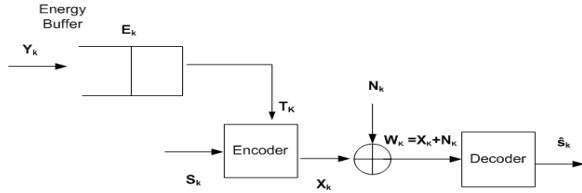


Fig. 1. The model

is true for many low quality, low rate sensor nodes ([2]). This assumption will be removed in Section IV. The sensor node is able to replenish energy by Y_k at time k . The energy available in the node at time k is E_k . This energy is stored in an energy buffer with an infinite capacity.

The node uses energy T_k at time k which depends on E_k and $T_k \leq E_k$. The process $\{E_k\}$ satisfies

$$E_{k+1} = (E_k - T_k)^+ + Y_k. \quad (1)$$

We will assume that $\{Y_k\}$ is stationary ergodic. This assumption is general enough to cover most of the stochastic models developed for energy harvesting. Often the energy harvesting process will be time varying (e.g., solar cell energy harvesting will depend on the time of day). Such a process can be approximated by piecewise stationary processes. As in [4], we can indeed consider $\{Y_k\}$ to be periodic stationary ergodic.

The encoder receives a message S from the node and generates an n -length codeword to be transmitted on the AWGN channel. The channel output $W_k = X_k + N_k$ where X_k is the channel input at time k and N_k is independent, identically distributed (*iid*) Gaussian noise with zero mean and variance σ^2 (we denote the corresponding Gaussian density by $\mathcal{N}(0, \sigma^2)$). The decoder receives $W^n \triangleq (W_1, \dots, W_n)$ and reconstructs S such that the probability of decoding error is minimized.

We will obtain the information-theoretic capacity of this channel. This of course assumes that there is always data to be sent at the sensor node. This channel is essentially different from the usually studied systems in the sense that the transmit power and coding scheme can depend on the energy available in the energy buffer at that time.

III. CAPACITY FOR THE IDEAL SYSTEM

In this section we obtain the capacity of the channel with energy harvesting node under ideal conditions of infinite energy buffer and energy consumption in transmission only.

The system starts at time $k = 0$ with an empty energy buffer and E_k evolves with time depending on Y_k and T_k . Thus $\{E_k, k \geq 0\}$ is not stationary and hence $\{T_k\}$ may also not be stationary. In this setup, a reasonable general assumption is to expect $\{T_k\}$ to be asymptotically stationary. One can further generalize it to be Asymptotically Mean Stationary (AMS), i.e.,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n P[T_k \in A] = \bar{P}(A) \quad (2)$$

exists for all measurable A . In that case \bar{P} is also a

probability measure and is called the *stationary mean* of the AMS sequence ([12]).

If the input $\{X_k\}$ is AMS, then it can be easily shown that for the AWGN channel $\{(X_k, W_k), k \geq 0\}$ is also AMS. Now the relevant channel capacity of our system is ([12])

$$C = \sup_{p_x} \bar{I}(X; W) = \sup_{p_x} \limsup_{n \rightarrow \infty} \frac{1}{n} I(X^n, W^n) \quad (3)$$

where $\{X_n\}$ is an AMS sequence, $X^n = (X_1, \dots, X_n)$ and the supremum is over all possible AMS sequences $\{X_n\}$. If $R < C$ then one can find a sequence of codewords with codelength n and rate R such that the average probability of error goes to zero as $n \rightarrow \infty$ ([12]).

In the following we obtain C in (3) for our system. We need the following definition. Although for C we need mutual information rate $\bar{I}(X; W)$, $I^*(X; W)$ defined below is easier to handle.

Pinsker Information Rate ([12]): Given an AMS random process $\{(X_n, W_n)\}$ with standard alphabets (Borel subsets of Polish spaces) A_X and A_W , the Pinsker information rate is defined as $I^*(X; W) = \sup_{q,r} \bar{I}(q(X); r(W))$ where the supremum is over all quantizers q of A_X and r of A_W and $\bar{I}(q(X); r(W)) = \limsup_{n \rightarrow \infty} I(q(X^n); r(W^n))/n$.

It is known that, $I^*(X; W) \leq \bar{I}(X; W)$. The two are equal if the alphabets are finite.

We also need the following Lemma for proving the achievability of the capacity of the channel. This Lemma holds for I^* but not for \bar{I} .

Lemma 1 ([12], Lemma 6.2.2): Let $\{(X_n, W_n)\}$ be AMS with distribution P and stationary mean \bar{P} . Then $I_P^*(X; W) = I_{\bar{P}}^*(X; W)$.

Theorem 1 For the energy harvesting system $C = 0.5 \log(1 + \frac{E[Y]}{\sigma^2})$.

Proof: Achievability: Let $\{X'_k\}$ be an *iid* Gaussian sequence with mean zero and variance $E[Y] - \epsilon$ where $\epsilon > 0$ is an arbitrarily small constant. We define the channel input $X_k = \text{sgn}(X'_k) \min(\sqrt{E_k}, |X'_k|)$ where $\text{sgn}(x) = 1$ if $x \geq 0$ and -1 if $x < 0$. Then $T_k = X_k^2 \leq E_k$ and $E[T_k] = E[X_k^2] \leq E[Y] - \epsilon$. Thus, from standard results on G/G/1 queues ([13], chapter 7) $E_k \rightarrow \infty$ *a.s.* and hence $|X_k - X'_k| \rightarrow 0$ *a.s.* Therefore, $\{X_k\}$ is AMS ergodic and $\{(X_k, W_k)\}$ is AMS ergodic.

By using Lemma 1 $I^*(X; W) = \sup_{q,r} \limsup_{n \rightarrow \infty} I(q(X^n); r(W^n))/n = I^*(X', W')$ where $I^*(X', W')$ corresponds to the limiting *iid* sequence $\{(X'_i, W'_i)\}$ where W'_i is the channel output corresponding to X'_i .

Also, since the mutual information between two random variables is the limit of the mutual information between their quantized versions ([14]), $I^*(X', W') = I(X', W') = 0.5 \log(1 + (E[Y] - \epsilon)/\sigma^2)$. Therefore, $I^*(X', W') \leq \bar{I}(X; W) = \limsup_{n \rightarrow \infty} I(X^n; W^n)/n$ and hence $\limsup_{n \rightarrow \infty} I(X^n; W^n)/n = \bar{I}(X; W) \geq 0.5 \log(1 + (E[Y] - \epsilon)/\sigma^2)$ for all $\epsilon > 0$.

Converse Part: For the system under consideration $\frac{1}{n} \sum_{k=1}^n T_k \leq \frac{1}{n} \sum_{k=1}^n Y_k \rightarrow E[Y]$ *a.s.* Hence, if $\{X_k(s), k = 1, \dots, n\}$ is a codeword for message $s \in \{1, \dots, 2^{nR}\}$ then for all large n we must

have $\frac{1}{n} \sum_{k=1}^n X_k(s)^2 \leq E[Y]$ with a large probability. Hence by the converse in the AWGN channel case, $\limsup_{n \rightarrow \infty} \frac{1}{n} I(X^n; W^n) \leq 0.5 \log(1 + E[Y]/\sigma^2)$.

Combining the direct part and converse part completes the proof. ■

Thus we see that the capacity of this channel is the same as that of a node with average energy constraint $E[Y]$, i.e., the hard energy constraint of E_k at time k does not affect its capacity. The capacity achieving signaling in the above theorem is *iid* Gaussian with zero mean and variance $E[Y]$.

If there is no energy buffer to store the harvested energy (Harvest-Use) then $T_k = X_k^2 \leq Y_k$. Thus $C = \max_{p_x} I(X; W) \leq 0.5 \log(1 + E[Y]/\sigma^2)$. The last inequality is strict unless X_k is $\mathcal{N}(0, E[Y])$. Then $X^2 = Y$ and hence Y_k is chi-square distributed with degree 1. If $Y_k \equiv E[Y]$ then the capacity will be that of an AWGN channel with peak and average power constraint $= E[Y]$. This problem is addressed in [15], [16], [17] and the capacity achieving distribution is finite and discrete. Let $X(y)$ denote a random variable having distribution that achieves capacity with peak power y . Then, for the general case the capacity of the channel is

$$C = E_Y[I(X(Y); W)]. \quad (4)$$

For small y , $X^2(y) = y$.

Thus, having energy buffer to store the harvested energy almost always strictly increases the capacity of the system under ideal conditions.

In [4], a system with a data buffer at the node which stores data sensed by the node before transmitting it, is considered. The stability region (for the data buffer) for the 'no-buffer' and 'infinite-buffer' corresponding to the harvest-use and harvest-store-use architectures are provided. Hence we see that the Shannon capacity achieving energy management policies provided here are close to the throughput optimal policies in [4]. Also the capacity is the same as the maximum throughput obtained in the data-buffer case in [4] for the infinite buffer architecture.

Next we comment on the capacity results when (1) represents E_{k+1} at the end of the k th slot where a slot represents m channel uses. In this case energy E_k is available not for one channel use but for m channel uses. This relaxes our energy constraints. Thus if $E[Y]$ still denotes mean energy harvested per channel use, then for infinite buffer case the capacity remains same as in Theorem 1.

IV. CAPACITY WITH PROCESSOR ENERGY (PE)

Till now we have assumed that all the energy that a node consumes is for transmission. However, sensing, processing and receiving (from other nodes) also require significant energy, especially in recent higher-end sensor nodes ([2]). We will now include the energy consumed by sensing and processing only.

We assume that energy Z_k is consumed by the node (if $E_k \geq Z_k$) for sensing and processing at time instant k . For transmission at time k , only $E_k - Z_k$ is available. $\{Z_k\}$ is assumed a stationary ergodic sequence. The rest of the system is as in Section II.

First we extend the achievable policy in Section III to incorporate this case. The signaling scheme $\{X_k\}$, $X_k = \text{sgn}(X'_k) \min(\sqrt{E_k}, |X'_k|)$ where $\{X'_k\}$ is *iid* Gaussian with zero mean and variance $E[Y] - E[Z] - \epsilon$ achieves the rate

$$R_{PE} = 0.5 \log \left(1 + \frac{E[Y] - E[Z] - \epsilon}{\sigma^2} \right). \quad (5)$$

If the sensor node has two modes: Sleep and Awake then the achievable rates can be improved. The sleep mode is a power saving mode in which the sensor only harvests energy and performs no other functions so that the energy consumption is minimal (which will be ignored). If $E_k < Z_k$ then we assume that the node will sleep at time k . But to optimize its transmission rate it can sleep at other times also. We consider a policy called *randomized sleep policy* in [6]. In this policy at every time instant k with $E_k \geq Z_k$ the sensor chooses to sleep with probability p independent of all other random variables. We will see that such a policy can be capacity achieving in the present context.

With the sleep option we will show that the capacity of this system is

$$C = \sup_{p_x: E[b(X)] \leq E[Y]} I(X; W) \quad (6)$$

where $b(x)$ is the cost of transmitting x and equals

$$b(x) = \begin{cases} x^2 + \alpha, & \text{if } |x| > 0, \\ 0, & \text{if } |x| = 0. \end{cases}$$

and $\alpha = E[Z]$. Observe that if we follow a policy that unless the node transmits, it sleeps, then b is the cost function. An optimal policy will have this characteristic. Denoting the expression in (6) as $C(E[Y])$, we can easily check that $C(\cdot)$ is a non-decreasing function. We also show below that $C(\cdot)$ is concave. These facts will be used in proving that (6) is the capacity of the system.

To show concavity, for $s_1, s_2 > 0$ and $0 \leq \lambda \leq 1$ we want to show that $C(\lambda s_1 + (1 - \lambda)s_2) \geq \lambda C(s_1) + (1 - \lambda)C(s_2)$. For s_i , let C_i be the capacity achieving codebook, $i = 1, 2$. Use λ fraction of time C_1 and $1 - \lambda$ fraction C_2 . Then the rate achieved is $\lambda C(s_1) + (1 - \lambda)C(s_2)$ while the average energy used is $\lambda s_1 + (1 - \lambda)s_2$. Thus, we obtain the inequality showing concavity.

Theorem 2 For the energy harvesting system with processing energy,

$$C = \sup_{p_x: E[b(X)] \leq E[Y]} I(X; W) \quad (7)$$

is the capacity for the system.

Proof: We prove this result in three steps. First we show the converse i.e., that the achievable rate cannot exceed (7). Next we show that this rate can be achieved by an *iid* signaling. Finally we show that we can achieve this rate by a signaling scheme that satisfies the energy constraints.

The converse follows via Fano's inequality as in, for example, [18], for an AWGN channel. For that proof to hold here, we need that $C(\cdot)$ is concave.

Next we show that an *iid* input sequence $\{X'_k\}$ satisfying $E[b(X)] = E[Y]$ achieves the capacity. This can be shown

in the usual way by using the following coding-decoding scheme.

Coding: Generate a code book of size n with codewords obtained from an *iid* sample with the capacity achieving distribution p_x with constraint $E[b(X)] = E[Y] - \epsilon$, where $\epsilon > 0$ is a small constant.

To transmit message m , take the corresponding codeword from the above codebook. If the codeword is ϵ -weakly typical and $\sum_{i=1}^n b(x_i)/n \leq E[Y] - \epsilon$, then transmit it; otherwise send an error message.

Decoding: On receiving W^n , the decoder finds the message m' which has its codeword jointly ϵ -weakly typical with W^n , if there is a unique such message. Otherwise it declares an error.

By the usual methods with the above coding-decoding scheme and also the fact that $C(\cdot)$ is non-decreasing, we can show that the probability of error for this scheme goes to zero as $n \rightarrow \infty$. Thus we can achieve the capacity (7).

Finally we show that we can achieve (7) by a signaling scheme satisfying the energy constraints. Take a codebook generated by *iid* sequences given above. Now define

$$X_k = \begin{cases} \min\{X'_k, \sqrt{(E_k - Z_k)^+}\}, & \text{if } X'_k \geq 0, \\ \max\{X'_k, -\sqrt{(E_k - Z_k)^+}\}, & \text{if } X'_k < 0. \end{cases}$$

Then to transmit X_k we need energy $T_k = (X_k^2 + Z_k)1_{\{X_k \neq 0\}}$ and $E_{k+1} = (E_k - T_k) + Y_k$. Also,

$$\begin{aligned} E[T_k] &= E[X_k^2] + E[Z_k]P\{X_k \neq 0\} \\ &\leq E[X_k'^2] + \alpha P\{X_k' \neq 0\} = E[Y] - \epsilon. \end{aligned}$$

Therefore, as in Theorem 1, $E_k \rightarrow \infty$ *a.s.* and hence $E_k - Z_k \rightarrow \infty$ *a.s.* and $|X_k - X'_k| \rightarrow 0$ *a.s.* Thus $\{X_k\}$ is AMS with the limiting process the *iid* sequence $\{X'_k\}$. Thus, as before, $\bar{I}(X; W) \geq I^*(X; W) = I^*(X'; W') = \bar{I}(X'; W')$. Therefore, we can achieve capacity in (7) by the input sequence $\{X_k\}$. ■

It is interesting to compute the capacity (7) and the capacity achieving distribution p_X . Without loss of generality, we can say that under p_X , with probability p ($0 \leq p \leq 1$) the node sleeps and with probability $1 - p$ it transmits with a density f that satisfies the cost function with equality. We can show using Kuhn-Tucker conditions that density f is

$$f(a) = \left(k_1 e^{-a^2 k_2} - \frac{p e^{-a^2/2\sigma^2}}{(1-p)\sqrt{2\pi\sigma^2}} \right)^+$$

where k_1 and k_2 are positive constants such that f satisfies the cost. When $E[Y]$ is large compared to $E[Z]$ f is close to $\mathcal{N}(0, E[Y] - E[Z])$.

Then density f can be computed numerically and in general it is not Gaussian.

To get further insight, let $\{B_k\}$ be *iid* binary random variables with $P[B_1 = 0] = p = 1 - P[B_1 = 1]$ and let $\{G_k\}$ be *iid* random variables with density f . Then $X'_k = B_k G_k$ is the capacity achieving *iid* sequence. Also,

$$\begin{aligned} I(X'_k; X'_k + N_k) &= h(B_k G_k + N_k) - h(N_k) \\ &= I(B_k; B_k G_k + N_k) + I(G_k; B_k G_k + N_k | B_k) \\ &= I(B_k; B_k G_k + N_k) + (1-p)I(G_k; G_k + N_k). \end{aligned}$$

This representation suggests the following interpretation (and coding theoretic implementation) of the scheme: the overall code is a superposition of a binary ON-OFF code and an *iid* code with density f . The position of the ON (and OFF) symbols is used to reliably encode $I(B; BG + N)$ bits of information per channel use, while the code with density f (which is used only during the ON symbols) reliably encodes $(1-p)I(G; G + N)$ bits of information per channel use.

In Fig.2 we compare the sleep-wake policies for $p = 0$ (equation (5)) and $p = 0.25$ with the optimal p . We take $E[Z] = 1$ and $\sigma^2 = 1$. We see that when $E[Y]$ is comparable

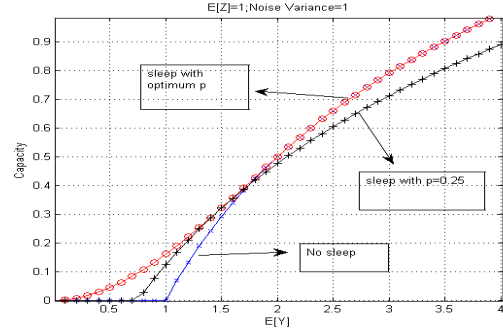


Fig. 2. Comparison of Sleep Wake policies

or less than $E[Z]$ then the node chooses to sleep with a high probability. When $E[Y] \gg E[Z]$ then the node will not sleep at all. Also it is found that when $E[Y] < E[Z]$, the capacity is zero when the node does not have a sleep mode. However we obtain a positive capacity if it is allowed to sleep.

V. ACHIEVABLE RATE WITH ENERGY INEFFICIENCIES

In this section we make our model more realistic by taking into account the inefficiency in storing energy in the energy buffer and the leakage from the energy buffer ([3]) for HSU architecture. For simplicity, we will ignore the energy Z_k used for sensing and processing.

We assume that if energy Y_k is harvested at time k , then only energy $\beta_1 Y_k$ is stored in the buffer and energy β_2 gets leaked in each slot where $0 < \beta_1 \leq 1$ and $0 < \beta_2 < \infty$. Then (1) becomes

$$E_{k+1} = ((E_k - T_k) - \beta_2)^+ + \beta_1 Y_k. \quad (8)$$

The energy can be stored in a super capacitor and/or in a battery. For a supercapacitor, $\beta_1 \geq 0.95$ and for the Ni-MH battery (the most commonly used battery) $\beta_1 \sim 0.7$. The leakage β_2 for the super-capacitor as well as the battery is close to 0 but for the super capacitor it may be somewhat larger.

In this case, similar to the achievability of Theorem 1 we can show that

$$R_{HSU} = 0.5 \log \left(1 + \frac{\beta_1 E[Y] - \beta_2}{\sigma^2} \right) \quad (9)$$

is achievable. This policy is neither capacity achieving nor throughput optimal [4]. An achievable rate of course is

(4). Now one does not even store energy and β_1, β_2 are not effective. The upper bound $0.5 \log(1 + E[Y]/\sigma^2)$ is achievable if Y is chi-square distributed with degree 1.

In *Harvest-Use* architecture since the energy harvested is used immediately, there is no loss due to storage inefficiency and leakage. Thus the achievable rate does not depend on β_1, β_2 . Hence, unlike Section III, the rate achieved by the HU may be larger than (9) for certain range of parameter values and distributions.

Another achievable policy for the system with an energy buffer with storage inefficiencies is to use the harvested energy Y_k immediately instead of storing in the buffer. The remaining energy after transmission is stored in the buffer. We call this *Harvest-Use-Store* (HUS) architecture. For this case, (8) becomes

$$E_{k+1} = ((E_k + \beta_1(Y_k - T_k)^+ - (T_k - Y_k)^+) - \beta_2)^+. \quad (10)$$

Compute the largest constant c such that $\beta_1 E[(Y_k - c)^+] > E[(c - Y_k)^+] + \beta_2$. This is the largest c such that taking $E[T_k] \leq c - \epsilon$ will make $E_k \rightarrow \infty$ a.s. Thus, as in Theorem 1, we can show that rate

$$R_{HUS} = 0.5 \log \left(1 + \frac{c}{\sigma^2} \right) \quad (11)$$

is achievable for this system. This is achievable by an input with distribution *iid* Gaussian with mean zero and variance c .

Equation (8) approximates the system where we have only rechargeable battery while (10) approximates the system where the harvested energy is first stored in a supercapacitor and after initial use transferred to the battery.

When $\beta_1 = 1, \beta_2 = 0$ we have obtained the capacity of this system in Section III. For the general case, its capacity is an open problem.

We illustrate the achievable rates mentioned above via an example.

A. Example 1

Let $\{Y_k\}$ be *iid* taking values in $\{0.25, 0.5, 0.75, 1\}$ with equal probability. We take the loss due to leakage $\beta_2 = 0$. In Figure 3 we compare the various architectures discussed in this section for varying storage efficiency β_1 . We use the result in [17] for computing the capacity in (4). From the

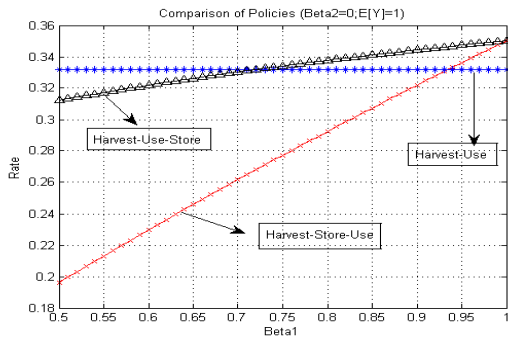


Fig. 3. Rates for various architectures

figure it can be seen that if the storage efficiency is very poor it is better to use the *HU* policy. This requires no

storage buffer and has a simpler architecture. If the storage efficiency is good *HUS* policy gives the best performance. For $\beta = 1$, the *HUS* policy and *HSU* policy have the same performance. Unlike the ideal system, the *HSU* (which uses infinite energy buffer) performs worse than the *HU* (which uses no energy buffer) when storage efficiency is poor.

Thus if we judiciously use a combination of a super capacitor and a battery, we may obtain a better performance.

VI. CONCLUSIONS

In this paper the Shannon capacity of an energy harvesting sensor node transmitting over an AWGN Channel is provided. It is shown that the capacity achieving policies are related to the throughput optimal policies. Also, the capacity is provided when energy is consumed in activities other than transmission. Achievable rates are provided when there are inefficiencies in energy storage.

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