

Queuing Theoretic and Information Theoretic Capacity of Energy Harvesting Sensor Nodes

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Abstract—Energy harvesting sensor networks provide near perpetual operation and reduce carbon emissions thereby supporting ‘green communication’. We study such a sensor node powered with an energy harvesting source. We obtain energy management policies that are throughput optimal. We also obtain delay-optimal policies. Next we obtain the Shannon capacity of such a system. Further we combine the information theoretic and queuing theoretic approaches to obtain the Shannon capacity of an energy harvesting sensor node with a data queue. Then we generalize these results to models with fading and energy consumption in activities other than transmission.

Keywords: Energy harvesting, sensor networks, data queue, throughput optimal policies, information-theoretic capacity, fading channel, processing energy.

I. INTRODUCTION

Network life time maximization ([1]) is an important design goal in battery powered wireless sensor network (WSN). A battery has a fixed amount of energy (ignoring the recharging and leakage effects) and it goes dead once the energy is exhausted. Hence the network life time gets linked to the node life time. Recently, energy harvesting techniques ([2], [3]) are gaining popularity as means of improving the network life time. An energy harvester harnesses energy from environment or other energy sources (e.g., body heat) and converts them to electrical energy. Common energy harvesting devices are solar cells, wind turbines and piezo-electric cells, which extract energy from the environment. Among these, harvesting solar energy through photo-voltaic effect seems to have emerged as a technology of choice for many sensor nodes ([3], [4]). One of the problems with generating electricity from solar and/or wind energy (as against the regular power supply or from a battery) is that the energy generated at any time is randomly changing with time ([5], [6]). For example, a solar cell will not generate any electricity in night. Thus, the communication system will need to carefully use and conserve its energy such that it is able to satisfy its requirements and is not starved of energy at any time.

A communication system with an energy harvesting source has an infinite amount of energy available over time but at a time only a random limited amount is available. Such a setup has not been studied much till recently ([6]- [7]). Thus the

new design criteria may be to match the energy generation profile of the harvesting source with the energy consumption profile of the sensor node. The energy consumption policy should be designed in such a way that the node can perform satisfactorily for a long time. In [2] such an energy/power management scheme is called *energy neutral operation*.

In this paper we address the issue of energy management using queuing theoretic and information theoretic formulations for a point to point channel such that the node is not starved of energy at any time. Next we combine information theory and queuing theory to provide a unified view of capacity for an energy harvesting sensor node with a data queue and transmitting over a Gaussian channel. Next we extend these techniques to fading channels and sleep-wake mechanisms. Our results are relevant for base stations with energy harvesting sources([8]).

Next we survey the related literature. Energy harvesting in sensor networks are studied in [9] and [10]. Conditions for energy neutral operation for various models of energy generation and consumption are provided in [2]. A practical solar energy harvesting sensor node prototype is described in [11]. In [12] the authors study optimal sleep-wake cycles for event detection probability in sensor networks.

The capacity of a fading Gaussian channel with channel state information (CSI) at the transmitter and receiver and at the receiver alone is provided in [13]. An excellent survey on fading channels is provided in [14]. In a recent contribution, optimal energy allocation policies over a finite horizon and fading channels are studied in [15].

Information capacity for a point to point Gaussian channel without leakage and processing energy is also available in [16]. Energy saving communications, modulation and MAC are studied in [17] and [18]. Relevant literature for models combining information theory and queuing theory are [19], [20] and [21].

The paper is organized as follows. Section II describes the energy harvesting model of a point to point channel. The energy harvested is stored in an energy queue and the data to be transmitted is stored in a data buffer. We first find the stability region of the data queue when the energy is spent only in transmission. We also provide delay optimal policies. These results are reported in [22]. Section III provides the Shannon capacity for a single node under the idealistic assumptions of

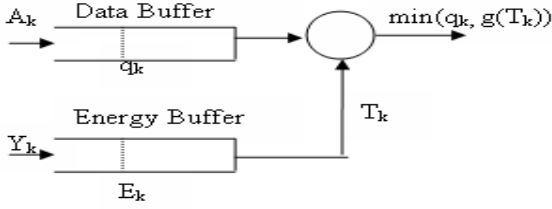


Fig. 1. Model of an energy harvesting point to point channel.

the data being available for transmission at all the time and the energy being spent in transmission only. These results are reported in [23]. Section IV obtains the information capacity of an energy harvesting sensor node with a data queue. Section V generalizes the results in Sections II and III to fading channels and when energy is consumed in activities other than transmission. Section VI concludes the paper.

II. MODEL AND NOTATION: QUEUING THEORETIC FORMULATION

We consider a transmitter (Fig. 1) generating packets to be transmitted to a receiver node. The system is slotted. During slot k (defined as time interval $[k, k + 1]$, i.e., a slot is a unit of time), A_k bits are generated. Although the transmitter may generate data as packets, we allow arbitrary fragmentation of packets during transmission. Thus, packet boundaries are not important and we consider bit strings. The bits A_k are eligible for transmission in $(k + 1)$ st slot. The queue length (in bits) at time k is q_k . The transmitter is able to transmit $g(T_k)$ bits in slot k if it uses energy T_k .

Initially, for simplicity, we assume that transmission consumes most of the energy at the transmitter and ignore other causes of energy consumption. This assumption is removed in Section V. We assume that the data buffer and the energy buffer are infinite. We denote by E_k the energy available at the node at time k . The energy harvesting source is able to replenish energy by Y_k in slot k . We initially assume that $\{A_k\}$ and $\{Y_k\}$ are *iid* but will generalize this assumption later.

The processes $\{q_k\}$ and $\{E_k\}$ satisfy

$$q_{k+1} = (q_k - g(T_k))^+ + A_k, \quad (1)$$

$$E_{k+1} = (E_k - T_k) + Y_k, \quad (2)$$

where $T_k \leq E_k$.

The function g is assumed to be monotonically non-decreasing. An important such function is given by Shannon's capacity formula $g(T_k) = \frac{1}{2} \log(1 + \beta T_k)$ bits per channel use for Gaussian channels where β is a constant such that βT_k is the SNR. This is a non-decreasing concave function. At low values of T_k , $g(T_k) \sim \beta_1 T_k$, i.e., g becomes a linear function. Thus in the following we limit our attention to linear and concave nondecreasing functions g . We also assume that $g(0) = 0$ which always holds in practice.

Many of our results (especially the stability results) will be valid when $\{A_k\}$ and $\{Y_k\}$ are (periodic) stationary,

ergodic. These assumptions are general enough to cover most of the stochastic models developed for traffic (e.g., Markov modulated) and energy harvesting.

In Subsection II-A we study the stability of this queue and identify easily implementable energy management policies which provide good performance.

A. Stability

We obtain a necessary condition for stability. Then we present a transmission policy which achieves the necessary condition, and show that it is also sufficient and is throughput optimal. The mean delay for this policy is not minimal. Thus, we obtain other policies which provide lower mean delay. In the next sub-section we consider delay optimal policies. These results are from [22] and [24].

Let us assume that we have obtained an (asymptotically) stationary and ergodic transmission policy $\{T_k\}$ which makes $\{q_k\}$ (asymptotically) stationary with the limiting distribution independent of q_0 . Taking $\{T_k\}$ asymptotically stationary seems to be a natural requirement to obtain (asymptotic) stationarity of $\{q_k\}$. The following Lemma gives a necessary condition for stability of such a policy.

Lemma 1: Let g be concave nondecreasing and $\{(A_k, Y_k)\}$ be stationary, ergodic sequences. For $\{T_k\}$ to be an asymptotically stationary, ergodic energy management policy that makes $\{q_k\}$ asymptotically stationary (jointly with $\{(T_k, A_k, Y_k)\}$) with a proper stationary distribution π it is necessary that $E[A_k] < E_\pi[g(T)] \leq g(E[Y])$. ■

Now, we present a policy that satisfies the above necessary condition. Let

$$T_k = \min(E_k, E[Y] - \epsilon) \quad (3)$$

where ϵ is a small positive constant with $E[A] < g(E[Y] - \epsilon)$. We show in Theorem 1 below that it is a throughput optimal policy. In other words, this policy keeps the queue stable if it is possible to do it by any (asymptotically) stationary, ergodic transmission policy.

Theorem 1: If $\{(A_k, Y_k)\}$ are stationary, ergodic and g is continuous, nondecreasing, concave then if $E[A_k] < g(E[Y_k])$, (3) makes the queue stable (with $\epsilon > 0$ such that $E[A] < g(E[Y] - \epsilon)$), i.e., it has a unique, stationary, ergodic distribution and starting from any initial distribution, q_k converges in total variation to the stationary distribution. ■

Thus, (3) is throughput optimal and we denote it by TO. Using this policy, the system operates in energy neutral operation and one need not know E_k and q_k exactly.

Although TO is a throughput optimal policy, it does not minimize mean delay. The Greedy policy

$$T_k = \min(E_k, f(q_k)) \quad (4)$$

where $f = g^{-1}$, looks promising ([24]). In Theorem 2, we will show that the stability condition for this policy is $E[A] < E[g(Y)]$ which is throughput optimal for linear g but strictly suboptimal for a strictly concave g . We will also show

in Section II-B that when g is linear, (4) also minimizes long term mean delay.

In next few results we assume that the energy buffer is finite, although large. For this case Lemma 1 and Theorem 1 also hold under the same assumptions.

Theorem 2: Let $\{A_k\}$, $\{Y_k\}$ be iid. If the energy buffer is finite, i.e., $E_k \leq \bar{e} < \infty$ (but \bar{e} is large enough) and $E[A] < E[g(Y)]$ then under the greedy policy (4), (q_k, E_k) has an Ergodic set. ■

The above result ensures that the Markov chain $\{(q_k, E_k)\}$ is ergodic and hence has a unique stationary distribution if $\{(q_k, E_k)\}$ is irreducible. A sufficient condition for this is $0 < P[A_k = 0] < 1$ and $0 < P[Y_k = 0] < 1$. In general, $\{(q_k, E_k)\}$ can have multiple ergodic sets. Then, depending on the initial state, $\{(q_k, E_k)\}$ converges to one of the ergodic sets and the limiting distribution depends on the initial conditions.

B. Delay Optimal Policies

In this section, we consider delay optimal policies. We choose T_k at time k as a function of q_k and E_k such that $E[\sum_{k=0}^{\infty} \alpha^k q_k]$ is minimized where $0 < \alpha < 1$ is a suitable constant. This minimizing policy is called α -discount optimal. When $\alpha = 1$, we minimize $\lim_{n \rightarrow \infty} \sup \frac{1}{n} E[\sum_{k=0}^{n-1} q_k]$. This optimizing policy is called average cost optimal. By Little's law ([25]) an average cost optimal policy also minimizes mean delay. If for a given (q_k, e_k) , the optimal policy T_k does not depend on the past values, and is time invariant, it is called a stationary Markov policy.

Theorem 3: Let $\{A_k\}$, $\{Y_k\}$ be iid with values in discrete sets or have probability densities. Also, let g be continuous and let the energy buffer be finite, i.e., $e_k \leq \bar{e} < \infty$. Then there exists an optimal α -discounted Markov stationary policy. If in addition $E[A] < g(E[Y])$ and $E[A^2] < \infty$, then there exists an average cost optimal stationary Markov policy. The optimal cost v does not depend on the initial state. Also, then the optimal α -discount policies tend to an optimal average cost policy as $\alpha \rightarrow 1$. Furthermore, if $v_\alpha(q, e)$ is the optimal α -discount cost for the initial state (q, e) then $\lim_{\alpha \rightarrow 1} (1 - \alpha) \inf_{(q, e)} v_\alpha(q, e) = v$. ■

In general one can compute a delay optimal policy numerically via Value Iteration or Policy Iteration. But this can be computationally intensive (especially for large data and energy buffer sizes). In sub-section II-A, we also provided a greedy policy (4) which is very intuitive, and is throughput optimal for linear g . However, for concave g (including the cost function $\frac{1}{2} \log(1 + \gamma t)$) it is *not* throughput optimal and provides low mean delays only for low load. However, we have the following.

Theorem 4: The Greedy policy (4) is α -discount optimal for $0 < \alpha < 1$ when $g(t) = \gamma t$ for some $\gamma > 0$. It is also average cost optimal. ■

III. MODEL AND NOTATION: INFORMATION THEORETIC FORMULATION

In this section we present an information theoretic model for an energy harvesting sensor node. We consider a sensor node

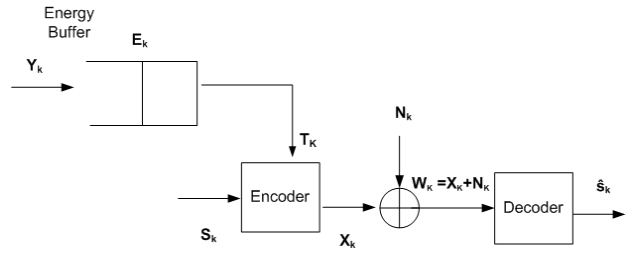


Fig. 2. The model: Information theoretic formulation

(Fig. 2) which is sensing and generating data to be transmitted to a central node via a discrete time AWGN Channel. As in Section II, we assume that transmission consumes most of the energy in a sensor node and ignore other causes of energy consumption. This assumption will be removed in Section V. The notation is as in II.

The node uses energy T_k at time k which depends on E_k and $T_k \leq E_k$. We assume that $\{Y_k\}$ is stationary, ergodic.

The encoder receives a message S from the node and generates an n -length codeword to be transmitted on the AWGN channel. The channel output $W_k = X_k + N_k$ where X_k is the channel input at time k and N_k is independent, identically distributed (*iid*) Gaussian noise with zero mean and variance σ^2 (we denote the corresponding Gaussian density by $\mathcal{N}(0, \sigma^2)$). The decoder receives $W^n \triangleq (W_1, \dots, W_n)$ and reconstructs S such that the probability of decoding error is minimized.

We will obtain the information-theoretic capacity of this channel. This of course assumes that there is always data to be sent at the sensor node. This channel is essentially different from the usually studied systems in the sense that the transmit power and coding scheme can depend on the energy available in the energy buffer at that time.

The system starts at time $k = 0$ with an empty energy buffer and E_k evolves with time depending on Y_k and T_k . Thus $\{E_k, k \geq 0\}$ is not stationary and hence $\{T_k\}$ may also not be stationary. In this setup, a reasonable general assumption is to expect $\{T_k\}$ to be asymptotically stationary. One can further generalize it to be Asymptotically Mean Stationary (AMS), i.e.,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n P[T_k \in B] = \bar{P}(B) \quad (5)$$

exists for all measurable B . In that case \bar{P} is also a probability measure and is called the *stationary mean* of the AMS sequence ([26]). We will further assume it to be ergodic.

If the input $\{X_k\}$ is AMS ergodic, then it can be easily shown that for the AWGN channel $\{(X_k, W_k), k \geq 0\}$ is also AMS ergodic. Now the relevant channel capacity of our system is ([26])

$$C = \sup_{p_x} \bar{I}(X; W) = \sup_{p_x} \limsup_{n \rightarrow \infty} \frac{1}{n} I(X^n, W^n) \quad (6)$$

where $\{X_n\}$ is an AMS ergodic sequence, $X^n = (X_1, \dots, X_n)$ and the supremum is over all possible AMS ergodic sequences

$\{X_n\}$. If $R < C$ then one can find a sequence of codewords with codelength n and rate R such that the average probability of error goes to zero as $n \rightarrow \infty$ ([26]).

In the following we obtain C in (6) for our system.

Theorem 5: For the energy harvesting system

$$C = 0.5 \log \left(1 + \frac{E[Y]}{\sigma^2} \right). \quad (7)$$

Thus we see that the capacity of this channel is the same as that of a node with average energy constraint $E[Y]$, i.e., the hard energy constraint of E_k at time k does not affect its capacity. The capacity achieving signaling in the above theorem is a codebook of *iid* Gaussian symbols X'_k with zero mean and variance $E[Y]$. The transmitted symbol $X_k = \text{sgn}(X'_k(s)) \min(\sqrt{E_k}, |X'_k(s)|)$ where $\text{sgn}(x) = 1$ if $x \geq 0$ and -1 if $x < 0$. Then $T_k = X_k^2 \leq E_k$ and $E[T_k] = E[X_k^2] \leq E[Y] - \epsilon^1$. Hence by using such a codebook the energy constraints are also satisfied.

IV. COMBINING INFORMATION AND QUEUING THEORY

In this section we consider a system with both energy and data buffer, each with infinite capacity. We use the notation q_k, A_k, Y_k as in Section II. There are n channel uses (mini slots) in a slot, i.e., the system uses an n length code to transmit data in a slot. As defined, Y_k, E_k and T_k are the energy generated, energy in the buffer and the energy spent in the k^{th} slot. We use codewords of length n and rate R_k in slot k with the following coding and decoding scheme.

- 1) An augmented message set $\{1, \dots, 2^{nR_k}\} \cup \{0\}$.
- 2) An encoder that assigns a codeword $x^n(m)$ to each $m \in \{1, \dots, 2^{nR_k}\} \cup \{0\}$ where $x^n(m)$ is generated as an *iid* sequence with distribution $\mathcal{N}(0, T_k/n - \delta_1)$ where $\delta_1 > 0$ is a small constant. The codeword $x^n(m)$ is retained if it satisfies the power constraint $\sum_i x_i^2 \leq T_k$. Otherwise an error message is sent.
- 3) A decoder that assigns a message $\hat{m} \in \{1, \dots, 2^{nR_k}\} \cup \{0\}$ to each received sequence y^n in a slot such that $(x^n(\hat{m}), y^n)$ is jointly weakly typical ([27]) and there is no other $x^n(m')$ jointly typical with y^n . Otherwise it declares an error.

In slot k , nR_k bits are taken out of the queue if $q_k \geq nR_k$. The bits are represented by a message $m_k \in \{1, \dots, 2^{nR_k}\}$ and $x^n(m_k)$ is sent. If $q_k < nR_k$ no bits are taken out of the queue and ‘‘0 message’’ $x^n(0)$ is sent.

Hence the processes $\{E_k\}$ and $\{q_k\}$ satisfy

$$\begin{aligned} q_{k+1} &= q_k - nR_k I_{\{q_k \geq nR_k\}} + A_k, \\ E_{k+1} &= (E_k - T_k) + Y_k. \end{aligned}$$

The rate R_k used in slot k is $0.5 \log(1 + T_k/\sigma^2)$ where T_k is chosen as in (3). Then $E_k \rightarrow \infty$ *a.s.* Thus, as in Section III, $T_k \rightarrow E[Y] - \epsilon$ *a.s.* Also, $R_k = 0.5 \log(1 + \frac{T_k}{\sigma^2}) \rightarrow 0.5 \log(1 + \frac{E[Y] - \epsilon}{\sigma^2})$. Thus we obtain

¹ $\epsilon > 0$ is an arbitrarily small constant. This is needed for energy neutral operation. Thus one will actually take $\{X'_k\}$ *iid* $\sim \mathcal{N}(0, E[Y] - \epsilon)$.

Theorem 6. The random data arrival process $\{A_k\}$ can be communicated with arbitrarily low average probability of block error, by an energy harvesting sensor node over a Gaussian channel, with a stable data queue, if and only if $E[A] < 0.5 \log(1 + \frac{E[Y]}{\sigma^2})$. ■

In Theorem 6 ‘stability’ of the queue has the following interpretation. If $\{A_k\}$ is stationary, ergodic then $P[q_k \rightarrow \infty] = 0$ and with probability 1, $\{q_k\}$ visits the set $\{q : q < nR\}$ infinitely often. Also the sequence $\{q_k\}$ is tight ([28]). If $\{A_k\}$ is *iid* then $\{q_k\}$ is a Markov chain and it has a finite number of ergodic sets. The process $\{q_k\}$ eventually enters one ergodic set with probability 1 and then approaches a stationary distribution. If $\{q_k\}$ is irreducible and aperiodic then $\{q_k\}$ has a unique stationary distribution and $\{q_k\}$ converges in distribution to it irrespective of initial conditions.

There has been a bit of variation in the modelling assumptions in Sections II, III and IV. In Section II the energy is varying on the time scale of a slot. In section III it is varying on the time scale of a mini slot. In the current Section again it is varying on the time scale of a slot. Which one is the right model depends on the practical system under consideration. Thus, in Theorem 5 we used the theoretical tool of AMS sequences. But in Theorem 6, in a slot we can use X_1, X_2, \dots, X_n *iid* Gaussian $\mathcal{N}(0, T_k/n - \delta)$ where δ is a small positive constant, and use a codeword only if it satisfies $X_1^2 + \dots + X_n^2 \leq T_k$; otherwise send an error message. Theorem 6 could also be formulated in the setup of Theorem 5. The conclusions will remain same.

V. EXTENSIONS

We consider generalizations of the models presented in Sections II and III. First we extend the results to the case of fading channels and then to the case where the processing energy at the transmitter node is non-negligible with respect to the transmission energy.

A. Fading Channel

We consider an AWGN fading channel in this section. In [22] this system has been studied in a queuing theoretic framework. It was shown that ‘water filling’ is throughput optimal for the Gaussian case. In [29] we have provided the information theoretic capacity. We have shown that, as for the unfaded case in Section III, the capacity is same as for the usual AWGN channel with fading when the channel state information is available at the transmitter and at the receiver. The capacity achieving distribution is zero mean Gaussian with the power allocation according to water filling. In this section we combine the information theoretic and the queuing theoretic results as in Section IV.

We use the notation in Section IV. In addition we assume that the channel experiences a flat, block fading $\{H_k\}$. The channel gain is constant over a slot. We assume the channel gain sequence $\{H_k\}$ to be stationary ergodic independent of $\{A_k\}, \{Y_k\}$. If the channel state information is available at the transmitter and receiver we use the ‘water filling’ power allocation policy in slot k

$$T(H) = \left(\frac{1}{H_0} - \frac{1}{H} \right)^+ \quad (8)$$

where H_0 is such that $E_H[T(H)] = E[Y] - \epsilon$ for some small $\epsilon > 0$. We use energy $T_k = \min(E_k, T(H_k))$. Then, $E_k \rightarrow \infty$ a.s. and $|T_k - T(H_k)| \rightarrow 0$ a.s. Therefore $R_k = 0.5 \log(1 + H_k^2 T_k / \sigma^2)$ and satisfies

$$\frac{1}{N} \sum_{k=1}^N R_k \rightarrow 0.5 E_H \left[\log \left(1 + \frac{H^2 T(H)}{\sigma^2} \right) \right] \text{ a.s.}$$

We have the following

Theorem 7. The random data arrival process $\{A_k\}$ can be communicated with arbitrarily low average probability of block error, by an energy harvesting sensor node over a fading Gaussian channel, with channel state information available at the transmitter and receiver, with a stable data queue if and only if $E[A] < 0.5 E_H[\log(1 + \frac{H^2 T(H)}{\sigma^2})]$. ■

B. Processing Energy

Till now we have assumed that all the energy that a node consumes is for transmission. However a node spends a considerable amount of energy for various other activities such as sensing and processing. In the following we take into account this energy. Now it is useful to have a sleep mode at the node. In the sleep mode the node only harvests energy. We assume that in this mode the node has to spend negligible amount of energy. If the node is awake in slot k then it senses data and generates A_k bits. It processes them (samples, quantizes, source codes and channel codes) and then stores in the data buffer. It may decide not to buffer the data generated with probability γ (then that data is lost). During these activities the node expends Z_k energy. We will assume $\{Z_k\}$ to be stationary, ergodic. It was shown in [30], that in this set up a good candidate for a throughput optimal policy is as follows. In slot k , the node sleeps with probability p independently of other events. Otherwise, it sleeps if $E_k < Z_k$. If awake, the node uses T_k energy for transmission where

$$T_k = \min \left(E_k - Z_k, \frac{E[Y]}{1-p} - E[Z_k] - \epsilon \right), \quad (9)$$

and $\epsilon > 0$ is a small constant. The p is selected to maximize

$$(1-p) \min \left((1-\gamma)E[A], g \left(\frac{E[Y]}{1-p} - E[Z] - \epsilon \right) \right),$$

and γ is chosen to satisfy

$$(1-p)(1-\gamma)E[A] = g \left(\frac{E[Y]}{1-p} - E[Z] - \epsilon \right) - \delta, \quad (10)$$

for a small positive δ . If we reduce δ , the throughput increases but the queuing delays also increase.

Information Theoretic Model: We have also obtained information theoretic capacity of the system when energy is spent in other activities. Suppose $\hat{Z}_i \geq 0$ is the energy spent in the i^{th} use of channel in the transmitter circuitry of the

node and T_i is the energy used in data transmission. We will also assume AWGN channel of Section III and for simplicity assume no fading. The rest of the set-up is of Section III. $\{\hat{Z}_i\}$ is assumed stationary, ergodic with $E[\hat{Z}_i] = \alpha \geq 0$. Now again, if $\alpha > 0$, it is useful to have the option to not transmit data with probability $q \geq 0$. In that case we will assume that no energy is spent in the transmitter circuitry, i.e., $\hat{Z}_i = 0$. Define the cost of transmitting x as

$$b(x) = \begin{cases} x^2 + \alpha, & \text{if } |x| > 0, \\ 0, & \text{if } |x| = 0. \end{cases}$$

Then the following capacity of the system is provided in [23].

Theorem 8 For the energy harvesting system with processing energy $\{\hat{Z}_i\}$ the capacity is,

$$C(E[Y]) = \sup_{p_x: E[b(X)] \leq E[Y]} I(X; W). \quad (11)$$

■
 C is a concave function of $E[Y]$. It is interesting to compute the capacity (11) and the capacity achieving distribution p_X . Without loss of generality, we can say that under p_X , with probability q ($0 \leq q \leq 1$) the node does not transmit and with probability $1-q$ it transmits with a density f that satisfies the cost function with equality. We can show using Kuhn-Tucker conditions that density h of the received symbol when the node is transmitting with density f is

$$h(a) = \left(k_1 e^{-a^2 k_2} - \frac{q e^{-a^2/2\sigma^2}}{(1-q)\sqrt{2\pi\sigma^2}} \right)^+ \quad (12)$$

where k_1 and k_2 are positive constants such that the cost constraint $E[b(X)] \leq E[Y]$ is satisfied. We have found that at the optimal q the term in (12) is always non-negative (thus $()^+$ is not needed at optimal q). This implies that whenever the node is transmitting, the density f is Gaussian with mean zero and variance $E[Y]/(1-q) - \alpha$.

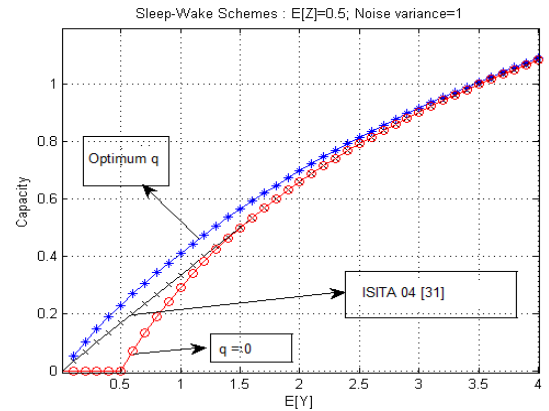


Fig. 3. Comparison of Sleep Wake policies.

The capacity achieving code is a superposition of a binary ON-OFF code (OFF means it is not transmitting) and an *iid* code with density f . Let $X = BG$. The position of the ON

(and OFF) symbols is used to reliably encode $I(B; BG + N)$ bits of information per channel use, while the code with density f (which is used only during the ON symbols) reliably encodes $(1 - q)I(G; G + N)$ bits of information per channel use. The capacity is plotted in Fig (3) for the optimal q along with the scheme provided in [31] and the policy with $q = 0$. We observe that, when $E[Y] \gg E[Z]$ there is no need to be OFF. However, when $E[Y] < E[Z]$ then the capacity without sleep is zero. We can obtain positive capacity with positive q . The improvement over the capacity in [31] is due the use of the superposition code.

Queuing Theoretic and Information Theoretic Model:

Now we combine the above two models of queuing theory and information theory. We assume that a node may sleep with probability p in a slot. In sleep mode it harvests only energy and hence expends negligible energy. If awake it generates A_k data and stores in the data buffer with probability $1 - \gamma$; otherwise it throws away the data. In the process, Z_k energy is spent in the slot. As in Section IV, there are n channel uses (mini slots) in a slot, i.e., the system uses an n length code to transmit the data in a slot. The node uses the above mentioned coding scheme which also has the option of not transmitting (if $\alpha > 0$) in a mini slot. Now the rate R_k used in slot k is (11) with $E[Y] = T_k$. With T_k in (9), $E_k \rightarrow \infty$ a.s. and hence R_k tends to right side of (10) a.s. with g the function C in (11) (since C is concave, it is continuous). Thus $\{q_k\}$ is stable in the sense of Section IV.

VI. CONCLUSIONS

We have considered a wireless communication system powered by energy harvesting sources. Throughput optimal and mean delay optimal energy management policies are identified which can make the system work in energy neutral operation. Also, often (when energy to transmission rate map g is linear) the greedy policy provided here is throughput as well as mean delay optimal. This does not require energy harvesting and traffic generation statistics and is easy to implement. Fundamental communication limits of such systems when the channel is AWGN is obtained. It is shown that the hard energy constraints does not affect the capacity. It is also shown that the capacity achieving policies are related to the throughput optimal policies. Next, the capacity of a system with energy harvesting sensor node with both data and energy queue is found out by combining the queuing theoretic and information theoretic approaches. The results are generalized to fading channels and when energy at the transmitter is also consumed in sensing and data processing.

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