Queued Cooperative Wireless Networks With Rateless Codes

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***Abstract*—Cooperative communication using rateless codes, in which the source transmits an infinite number of parity bits to the destination until the receipt of an acknowledgment, has recently attracted considerable interest. It provides a natural and efficient mechanism for accumulating mutual information from multiple transmitting relays. We develop an analysis of queued cooperative relay systems that combines the communication-theoretic transmission aspects of cooperative communication using rateless codes over Rayleigh fading channels with the queuing-theoretic aspects associated with buffering messages at the relays. Relay cooperation combined with queuing reduces the message transmission times and also helps distribute the traffic load in the network, which improves throughput significantly.**

***Index Terms*—cooperative communications, queuing, relays, selection, rateless codes, fading channels**

I. Introduction

Cooperative communications, where different nodes in a network work together to transmit information from a source to a destination, decreases energy expenditure and improves the reliability of data transmission in a wireless network. Given its promising gains, it has attracted considerable attention in the last few years [1]–[5]. A variety of cooperation schemes have been proposed such as cooperative beamforming, distributed space-time codes, amplify and forward, decode and forward, dynamic decode and forward, etc.

Fountain codes, and, in general, rateless codes have recently been shown to be well suited for cooperative relay networks [6]–[8]. Unlike conventional codes, which generate a finite number of parity bits, rateless codes generate an infinite number of parity bits, which are transmitted until an acknowledgment is received from the recipient. Rateless codes provide a natural way for accumulating information from one or more relays. With them, a receiver can recover the original information from *unordered* subsets of *one or more* rateless code-streams transmitted by multiple sources so long as the total mutual information accumulated marginally exceeds the entropy of the source information [9]–[12].

In [6], an asynchronous mode was considered, in which a relay starts transmitting to the destination a fountain-encoded stream of the messages as soon as it decodes the message. This enables the remaining relays and the destination to decode the source message faster. In [7], a rateless coding framework for space-time collaboration over a single relay channel was proposed. In [8], performance gains were shown when the cooperative relay used low-density generator-matrix codes, and directly transmitted to the destination some or all of the noise-corrupted coded bits it received from the source.

All the above papers approach the problem from a communication-theoretic perspective, in which the goal is to have the message reach the destination as soon as possible or as reliably as possible. The source transmits the next message only after the current message has been received by the destination. In practice, it is more efficient for the source to start transmitting its next message earlier, while the current one is en route to the destination, and to buffer/queue messages at intermediate relays. Such queues arise not just for full-duplex relays, which can transmit and receive simultaneously, but also for half-duplex relays that cannot. Consequently, the performance of the cooperative relay system and its end-to-end message delay depends not only on the message transmission times but also the queuing delays at the relays.

This paper develops an analysis of such queued cooperative relay systems, which combines the communication-theoretic transmission aspects of cooperative communication using rateless codes over fading channels with the queuing-theoretic aspects associated with buffering at the relays. We analyze the performance of the following two cooperation schemes: (i) relay selection diversity, in which the relay with the best instantaneous link to the source first receives the message, and forwards it to the destination; and (ii) relay cooperation with strong inter-relay links. The latter case is insightful as it is an extreme case of the asynchronous scenario mentioned above, and it models the scenario when all the relays cooperate simultaneously. For all of these schemes we derive the average throughput, stability region of the relay queues, and the average end-to-end time of a message. To benchmark the benefits of cooperation, we also develop corresponding results for two traditional queued relay models. Our analysis also explicitly incorporates transmitter time-out, which, as we show, is necessary for transmitting rateless codes over Rayleigh fading channels.

The outline of the paper is as follows. We describe the system model in Sec. II, and develop the analysis in Sec. III. Simulation results are presented in Sec. IV, and are followed by our conclusions in Sec. V.

II. System Model

As shown in Fig. 1, we consider a two-hop network in which the source, $\mathcal{S}$, has a continuous stream of messages to transmit to the destination, $\mathcal{D}$, via M decode-and-forward relays, $\mathcal{R}_1, \ldots, \mathcal{R}_M$. Each message has a bandwidth-normalized payload of B nats/Hz. The signal transmitted by a source is received by multiple relays due to the broadcast nature of the wireless channel.

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The wireless channels between the different nodes are assumed to be frequency-flat, block-fading Rayleigh channels. The channel is assumed to be constant over the duration of transmission of the message; it changes to an independent value thereafter [6]. For rateless codes, this assumption is valid in low to medium mobility scenarios of current high data rate wireless systems, or when the time out period is less than the channel coherence time. This analytically tractable model is commonly used in the cooperative rateless code literature mentioned above and, in general, in the analysis of adaptive rate systems with fixed message payloads.[1] To simplify the theoretical treatment, we also assume that the direct source-destination link is very weak. The source-relay (SR) channel gains and the relay-destination (RD) channel gains for different relays are assumed to be independent, as is typically the case.

The source as well as the relays use different rateless codes to transmit their messages. When a relay receives a sufficient number of bits, it can successfully decode the message, which is then queued in the relay's buffer for transmission in a first come first serve fashion to the destination. The minimum time taken by relay i to decode a message from $\mathcal{S}$ is $(1+\delta)\frac{B}{\log_e(1+\gamma_i)}$, where γ_i is the receive SNR for the $\mathcal{S}$–$\mathcal{R}_i$ link, and δ is the inefficiency of the practical implementation of the rateless code [6]. Henceforth, we ignore δ as it can be factored into B. From the Rayleigh fading assumption, it follows that γ_i and λ_i, which denotes the receive SNR for the $\mathcal{R}_i$–$\mathcal{D}$ link, are exponentially distributed.

In order for the receiver to separate the various transmissions, we assume that the relays and the source transmit their signals using different, a priori assigned, spreading sequences. It has been shown in [6] that a typical system can operate well with low spreading gains. Such an approach enables different information streams to be transmitted in a flexible and decentralized way, and still be distinguishable at the receiver(s). For analytical tractability, we place no limitations on the number of such signals that the receiver can separate. In practice, this is limited by the number of despreaders and decoders available at the receiver. We assume an infinitely long buffer at each relay, to keep the analysis simple.

No channel knowledge is assumed at the transmitters. The transmitting node will time out if no acknowledgment is received within a specified time, t_{out}. Time out is necessary for transmitting a rateless code over a block-fading Rayleigh fading channel since the average transmission time is otherwise infinite.[2] In the event of a time out, the transmitting node can either drop the message from its queue or retransmit the message. Due to space constraints, only message dropping is analyzed in this paper.

The relays are assumed to be full-duplex capable. This is practically feasible using a duplexer when the carrier frequency used for the SR channels is sufficiently apart from that for the RD channels. The analysis for the half-duplex case is slightly different since the transmission and reception become mutually exclusive; it is not presented in this paper due to space constraints.

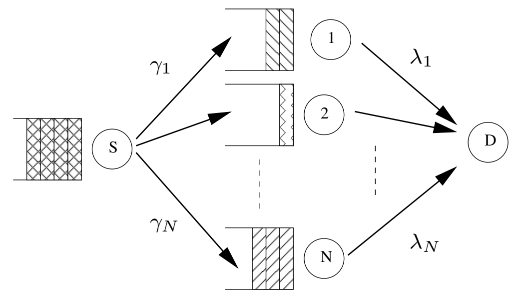

Fig. 1. Schematic of two hop queued cooperative relay network

Several possibilities exist for the source to transmit the message using rateless codes and relays. We first analyze queued relay selection diversity. Since the asynchronous scheme of [6] is analytically intractable even without queuing, we analyze the queued strong inter-relay link case, which provides a bound on its performance. Also considered are two traditional queued relaying schemes. For all of them, we derive closed-form analytical expressions for the throughput, relay queue stability regions, and the overall end-to-end delay in the network.

A. Queued Cooperative Transmission Schemes

The various schemes are briefly described below.

Queued relay selection (RS) diversity network: In this scheme, the source transmits until *any one* of the relays decodes the message and acknowledges back. The source then starts transmitting the next message. The relay queues the message in its buffer for transmission to the destination; the other relays play no further role in the transmission of the message. This scheme thus automatically selects the relay with the best instantaneous SR link.[3]

Queued strong inter-relay network: In this scheme, the inter-relay links are so strong that once a relay decodes a message, the other relays also get the message by listening to that relay's transmission to the destination. Consequently, the destination accumulates information from all the relays' transmissions, and quickly decodes the message.

Traditional queued relay networks: Traditional models do not exploit spatial diversity through cooperation – the source selects the relay node to transmit its message to (using a rateless code) either probabilistically or in a round-robin (RR) fashion. Once the intended relay decodes the message, it sends an acknowledgment back to the source, and stores the message in its queue for transmission to the destination. The source then starts transmitting the next message.

B. Notation

For a random variable (RV) Y, $Y[k]$, $k = 1, 2, \ldots$, will denote an i. i. d. sequence with the distribution of Y. $\mathbf{E}[.]$ and

[1]In practice, when the channel gain is very high and the transmission durations are small, successive messages sent by the same node will experience correlated channels.

[2]Without time out, the average time to transmit a message is $\int_0^\infty \frac{B}{\bar{h}\log(1+h)}\exp(-h/\bar{h})dh$, where $\bar{h} = \mathbf{E}[h]$. Using variable transformations, this equals $\int_0^\infty \frac{B}{y\bar{h}}\exp\left(-(e^y-1)/\bar{h}\right)dy$. The integrand is unbounded at $y = 0$, and so is the integral.

[3]Mechanisms that also account for the RD link state can be also implemented, but are beyond the scope of this paper.

Var $[.]$ shall denote the expectation and variance, respectively. We shall interchangeably use $\mathbf{E}\left[Y[k]^m\right]$ and $\mathbf{E}\left[Y^m\right]$. The cumulative distribution function (CDF) of Y will be denoted by $F_Y(.)$.

Let S^{SR} be the time taken to transmit a message from the source (including those that are dropped). Let S_i^{SR} denote a message transmission time given that relay i will receive the message (unless the source times out). Let A_i denote the inter-arrival time between messages that (successfully) arrive at relay i. Let $S_{\text{nd}}^{\text{SR}}$ denote the transmission time of a message given that it is not dropped, i.e., $S_{\text{nd}}^{\text{SR}} = \left[S^{\text{SR}} | S^{\text{SR}} < t_{\text{out}}\right]$. Similarly, S_i^{RD} is the message transmission time by relay i (including messages that are dropped). And, $S_{\text{nd},i}^{\text{RD}}$ is the corresponding time given that the message is not dropped by relay i. Relay i's queue is denoted by $\mathcal{Q}_i$. The probability that the source times out when transmitting is denoted by $P_{\text{out}}^{\text{SR}}$. Borrowing terminology from queuing theory, we shall interchangeably use the terms 'message transmission time' and 'service time'. Also, let $X_i^{\text{SR}} = 1/\log_e(1+\gamma_i)$ and $X_i^{\text{RD}} = 1/\log_e(1+\lambda_i)$ denote the reciprocals of the SR and RD transmission rates. Let $\rho_{\text{SR}} = \mathbf{E}\left[\gamma_i\right]$ and $\rho_{\text{RD}} = \mathbf{E}\left[\lambda_i\right]$.

C. Relevant Queuing Theory Results

We state below some relevant results for the GI/GI/1 queues that we shall encounter in the paper. In GI/GI/1 queues, the inter-arrival time of messages and the service time are i. i. d. with general distributions.

Stability: A queue with inter-arrival time of messages given by the RV A and service time per message given by the RV S is stable if and only if[4] ([13], [14])

$$\mathbf{E}\left[A\right] > \mathbf{E}\left[S\right]. \tag{1}$$

Waiting time: For a general GI/GI/1 queue, an exact expression for the mean waiting time is not available in literature. However, one approximation, which works reasonably well, especially under heavy traffic, is [15]

$$\mathbf{E}\left[W\right] \approx \frac{\eta g \mathbf{E}\left[S\right]\left(C_S^2 + C_A^2\right)}{2(1-\eta)}, \tag{2}$$

where $\eta = \frac{\mathbf{E}[S]}{\mathbf{E}[A]}$, $C_Y^2 = \frac{\text{Var}[Y]}{\mathbf{E}[Y]^2}$, for an RV Y. If $C_A^2 < 1$, $g = \exp\left[\frac{-2(1-\eta)(1-C_A^2)^2}{3\eta(C_A^2+C_S^2)}\right]$, else, for $C_A^2 \geq 1$, $g = \exp\left[-(1-\eta)\frac{C_A^2-1}{C_A^2+4C_S^2}\right]$.

III. Analysis

We will assume that the SR links are i. i. d., and so are the RD links. The analysis is similar, albeit more cumbersome, for the general non-identical links case.

A. Queued Relay Selection Diversity Network

In this case, the relay with the highest SR channel gain will receive the message first.

[4]By stable, we mean that the queue length and waiting time processes have unique proper stationary distributions; starting from any initial condition, these processes converge weakly to the stationary distributions.

1) Statistics of S^{SR}: Given that the various SR links are independent, we get the following if there was no provision of time out:

$$F_{S^{\text{SR}}}(x) = 1 - \prod_{i=1}^{M} P\left(\log_e(1+\gamma_i) < \frac{B}{x}\right), \tag{3}$$

$$= 1 - \left(1 - \exp\left[-\frac{1}{\rho_{\text{SR}}}\left(e^{B/x} - 1\right)\right]\right)^M. \tag{4}$$

Therefore, the probability that the source times out and drops the message is

$$P_{\text{out}}^{\text{SR}} = \left(1 - \exp\left[-\frac{1}{\rho_{\text{SR}}}\left(e^{B/t_{\text{out}}} - 1\right)\right]\right)^M. \tag{5}$$

From (4), it can be also shown that for $0 \leq x < t_{\text{out}}$

$$p_{S_{\text{nd}}^{\text{SR}}}(x) = \frac{1}{1 - P_{\text{out}}^{\text{SR}}} \frac{BM}{\rho_{\text{SR}} x^2} \times \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \exp\left[\frac{B}{x} - \frac{k+1}{\rho_{\text{SR}}}\left(e^{B/x} - 1\right)\right], \tag{6}$$

with $p_{S_{\text{nd}}^{\text{SR}}}(x) = 0$, for $x \geq t_{\text{out}}$.

2) Statistics of inter-arrival time A_i: In general, the inter-arrival time at $\mathcal{R}_i$ with message dropping can be written as

$$A_i = S_{\text{nd},i}^{\text{SR}} + \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{k=1}^{N_j[1]+N_j[2]+\cdots+N_j[c]} S_j^{\text{SR}}[k] + (c-1)t_{\text{out}}, \tag{7}$$

where c is the number of cycles between two *successful* message arrivals in $\mathcal{Q}_i$. A cycle is the inter-arrival time between consecutive epochs in which relay i is the best relay, i.e, $X_i^{\text{SR}} < X_j^{\text{SR}}$, for all $j \neq i$. In the lth cycle, let $N_j[l]$ denote the number of times relay j is the best relay. Then, the lth cycle duration equals $\sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{k=1}^{N_j} S_j^{\text{SR}} + S_{\text{nd},i}^{\text{SR}}$ if the message is successfully received by relay i at the end of the cycle. Otherwise, it equals $\sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{k=1}^{N_j} S_j^{\text{SR}} + t_{\text{out}}$, if time out occurs when $\mathcal{R}_i$ is the best relay. Clearly, $P(c) = (P_{\text{out}}^{\text{SR}})^{c-1} P_{\text{out}}^{\text{SR}}$, since the source times out with probability $P_{\text{out}}^{\text{SR}}$ in the first $c-1$ cycles, but not the last one. Its moments are given by the following Lemma.

Lemma 1: The mean inter-arrival time of messages at relay i in relay selection is

$$\mathbf{E}\left[A_i\right] = M\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right] + Mt_{\text{out}} \frac{P_{\text{out}}^{\text{SR}}}{1 - P_{\text{out}}^{\text{SR}}}. \tag{8}$$

The variance of the inter-arrival time is

$$\begin{aligned} \text{Var}\left[A_i\right] = &\left(t_{\text{out}} + (M-1)\mathbf{E}\left[S^{\text{SR}}\right]\right)^2 \text{Var}\left[c\right] + \text{Var}\left[S_{\text{nd}}^{\text{SR}}\right] \\ &+ (M-1)(M-2)\mathbf{E}\left[S^{\text{SR}}\right]^2 \mathbf{E}\left[c^2\right] \\ &+ (M-1)\mathbf{E}\left[c\right]\left(\mathbf{E}\left[S^{\text{SR}}\right]^2 + \mathbf{E}\left[{S^{\text{SR}}}^2\right]\right). \end{aligned} \tag{9}$$

We now evaluate the constituent terms, $\mathbf{E}\left[A_i\right]$ and $\mathbf{E}\left[A_i^2\right]$, in (9). From (6), it can be shown for $m = 1$ and 2 that

$$\mathbf{E}\left[\left(S_{\text{nd}}^{\text{SR}}\right)^m\right] = \frac{1}{1 - P_{\text{out}}^{\text{SR}}} \frac{B^m M}{\rho_{\text{SR}}} \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \psi_m\left(\frac{\rho_{\text{SR}}}{k+1}, \frac{B}{t_{\text{out}}}\right), \tag{10}$$

where, for $a > 0$, $u > 0$, and $m \geq 1$,

$$\psi_m(a,u) = \int_u^\infty \frac{1}{x^m} \exp\left(x + \frac{1-e^x}{a}\right) dx. \quad (11)$$

(The intermediate steps are not shown to conserve space.) Furthermore, it can be shown for $m = 1, 2$ that

$$\mathbf{E}\left[\left(S^{\text{SR}}\right)^m\right] = (1 - P_{\text{out}}^{\text{SR}})\mathbf{E}\left[\left(S_{\text{nd}}^{\text{SR}}\right)^m\right] + P_{\text{out}}^{\text{SR}} t_{\text{out}}^m. \quad (12)$$

3) Statistics of message transmission times at relay i: The message transmission times at relay i are an i. i. d. sequence. Similar to the SR link analysis, the probability distribution of $S_{\text{nd},i}^{\text{RD}}$ is given by

$$p_{S_{\text{nd},i}^{\text{RD}}}(s) = \frac{B}{\rho_{\text{RD}} s^2 (1 - P_{\text{out}}^{\text{RD}})} \exp\left[\frac{B}{s} - \frac{1}{\rho_{\text{RD}}}\left(e^{B/s} - 1\right)\right], \quad 0 \leq s < t_{\text{out}}, \quad (13)$$

where $P_{\text{out}}^{\text{RD}}$ is the probability that the relay times out when it is transmitting to the destination. The following results follow from (13):

$$P_{\text{out}}^{\text{RD}} = 1 - \exp\left[-\frac{1}{\rho_{\text{RD}}}\left(e^{B/t_{\text{out}}} - 1\right)\right], \quad (14)$$

$$\mathbf{E}\left[\left(S_i^{\text{RD}}\right)^m\right] = P_{\text{out}}^{\text{RD}} t_{\text{out}}^m + \frac{B^m}{\rho_{\text{RD}}} \psi_m\left(\rho_{\text{RD}}, \frac{B}{t_{\text{out}}}\right). \quad (15)$$

Since the inter-arrival times and the message transmission times at $\mathcal{Q}_i$ are i. i. d., it is a GI/GI/1 queue. Combining the above results with (1) gives the following result.

Theorem *1:* The relay queues in relay selection are stable if and only if

$$P_{\text{out}}^{\text{RD}} t_{\text{out}} + \frac{B}{\rho_{\text{RD}}} \psi_1\left(\rho_{\text{RD}}, \frac{B}{t_{\text{out}}}\right) < M t_{\text{out}} \frac{P_{\text{out}}^{\text{SR}}}{1 - P_{\text{out}}^{\text{SR}}} + \frac{1}{1 - P_{\text{out}}^{\text{SR}}} \frac{BM^2}{\rho_{\text{SR}}} \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \psi_1\left(\frac{\rho_{\text{SR}}}{k+1}, \frac{B}{t_{\text{out}}}\right). \quad (16)$$

Expressions for the average delay in $\mathcal{Q}_i$ and the end-to-end time now follow by substituting the above results in (2). The throughput of the system can be shown to be

$$\Lambda = M \frac{1 - P_{\text{out}}^{\text{RD}}}{\max\left(\mathbf{E}\left[S_i^{\text{RD}}\right], \mathbf{E}\left[A_i\right]\right)}, \quad (17)$$

where, due to symmetry, i indexes an arbitrary relay. This expression includes the scenario where the relay queues are unstable. The average end-to-end time for a message that reaches the destination is

$$\xi = \mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right] + \mathbf{E}\left[W_i\right] + \mathbf{E}\left[S_{\text{nd},i}^{\text{RD}}\right], \quad (18)$$

where W_i is the waiting time of a message in $\mathcal{Q}_i$, and is evaluated by substituting the above results in (2).

B. Traditional Queued Relay Network

In traditional relay networks, the source transmits a message to only one of the relays, which is decided before the transmission starts. We consider the probabilistic selection (PS) and RR policies, described in Sec. II-A. In PS, the source randomly chooses relay i, with probability $p_i = 1/M$. In RR, the source sequentially transmits to all relays sequentially.

1) Statistics of S^{SR} for RR and PS: For both PS and RR, the CDF of message transmission time to $\mathcal{R}_i$ is $F_{S^{\text{SR}}}(x) = P\left(X_i^{\text{SR}} < \frac{x}{B}\right)$. It follows that the source time out probability is

$$P_{\text{out}}^{\text{SR}} = 1 - \exp\left[-\frac{1}{\rho_{\text{SR}}}\left(e^{B/t_{\text{out}}} - 1\right)\right]. \quad (19)$$

And, the probability distribution of $S_{\text{nd}}^{\text{SR}}$ is

$$p_{S_{\text{nd}}^{\text{SR}}}(s) = \frac{1}{1 - P_{\text{out}}^{\text{SR}}} \frac{B}{\rho_{\text{SR}} s^2} \exp\left[\frac{B}{s} - \frac{1}{\rho_{\text{SR}}}\left(e^{B/s} - 1\right)\right]. \quad (20)$$

In effect, this is the same as (6) with $M = 1$ since the relay is selected without regard to the channel condition.

2) Statistics of inter-arrival time A_i for PS: For PS, the general expression for the inter-arrival time at $\mathcal{R}_i$ is given by

$$A_i = S_{\text{nd},i}^{\text{SR}} + \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{k=1}^{N_j[1]+N_j[2]+\cdots+N_j[c]} S_j^{\text{SR}}[k] + (c-1) t_{\text{out}}. \quad (21)$$

The reasoning behind this is similar to that for relay selection, and is not repeated here. As before, $P(c) = (P_{\text{out}}^{\text{SR}})^{c-1} P_{\text{out}}^{\text{SR}}$, where $P_{\text{out}}^{\text{SR}}$ is given by (19). Therefore, $\mathbf{E}\left[A_i\right]$ and $\text{Var}\left[A_i\right]$ are as per Lemma 1, albeit with different parameter values.

3) Statistics of inter-arrival time A_i for RR: The inter-arrival time expression for RR is different from PS because the source transmits to the relays sequentially. It takes the form

$$A_i = S_{\text{nd},i}^{\text{SR}} + \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{k=1}^{c} S_j^{\text{SR}}[k] + (c-1) t_{\text{out}}. \quad (22)$$

This expression is similar to (21), except that each relay is visited *exactly once* in RR in every cycle, i.e., $N_j[k] = 1$, $1 \leq k \leq c$. While $\mathbf{E}\left[A_i\right]$ for RR and PS turns out to be the same (and as per (8)), $\text{Var}\left[A_i\right]$ is different and takes the form

$$\text{Var}\left[A_i\right] = \text{Var}\left[S_{\text{nd},i}^{\text{SR}}\right] + (M-1)\mathbf{E}\left[c\right] \text{Var}\left[S_i^{\text{SR}}\right] + \left(t_{\text{out}} + (M-1)\mathbf{E}\left[S_i^{\text{SR}}\right]\right)^2 \text{Var}\left[c\right]. \quad (23)$$

We now evaluate the terms that constitute the above expressions. As for relay selection, we get for both PS and RR

$$\mathbf{E}\left[\left(S_{\text{nd}}^{\text{SR}}\right)^m\right] = \frac{1}{1 - P_{\text{out}}^{\text{SR}}} \frac{B^m}{\rho_{\text{SR}}} \psi_m\left(\rho_{\text{SR}}, \frac{B}{t_{\text{out}}}\right). \quad (24)$$

The $\mathbf{E}\left[S^{\text{SR}}\right]$ and $\mathbf{E}\left[{S^{\text{SR}}}^2\right]$ expressions then follow using (12).

4) Statistics of service time at relay i for RR and PS: Clearly, the probability distribution of the relay transmission time for PS and RR is the same as for the relay selection case. Its first and second moments are therefore given by (15). The probability, $P_{\text{out}}^{\text{RD}}$, that the relay times out is given by (14).

For both PS and RR, since the inter-arrival times and the message transmission times at each relay are i. i. d., it again follows that $\mathcal{Q}_i$ is a GI/GI/1 queue.

Theorem *2:* The relay queues in round-robin and probabilistic selection are stable if and only if

$$P_{\text{out}}^{\text{RD}} t_{\text{out}} + \frac{B}{\rho_{\text{RD}}} \psi_1\left(\rho_{\text{RD}}, \frac{B}{t_{\text{out}}}\right) < M t_{\text{out}} \frac{P_{\text{out}}^{\text{SR}}}{1 - P_{\text{out}}^{\text{SR}}} + \frac{1}{1 - P_{\text{out}}^{\text{SR}}} \frac{BM}{\rho_{\text{SR}}} \psi_1\left(\rho_{\text{SR}}, \frac{B}{t_{\text{out}}}\right). \quad (25)$$

The average throughput, message waiting time in $\mathcal{Q}_i$ and end-to-end delay are obtained by substituting the above expressions in (17), (2), and (18), respectively.

C. Queued Relay Network with Strong Inter-Relay Links

In this set up, all M relays receive the message from the source at the same time, and also start and stop transmitting it to the destination simultaneously. In effect, the relays together behave as one relay $\mathcal{R}_{\text{eq}}$ with an equivalent queue $\mathcal{Q}_{\text{eq}}$.

1) Statistics of S^{SR}: The message is received by all the relays when any one of them first gets it, which is the same as for relay selection case. Therefore, the probability distribution of $S_{\text{eq}}^{\text{SR}}$, its first and second moments, and $P_{\text{out}}^{\text{SR}}$ are as derived in Sec. III-A.

2) Statistics of inter-arrival time A_{eq}: The inter-arrival time at $\mathcal{Q}_{\text{eq}}$ now equals

$$A_{\text{eq}} = S_{\text{nd,eq}}^{\text{SR}} + w t_{\text{out}}, \quad (26)$$

where w is the number of time outs that occur between two arrivals at $\mathcal{R}_{\text{eq}}$. Clearly, $P(w = m) = (P_{\text{out}}^{\text{SR}})^m (1 - P_{\text{out}}^{\text{SR}})$. The expressions for the mean and variance of A_{eq} follow by substituting $M = 1$ in (8) and (9), respectively.

3) Statistics of service time at equivalent relay: Since the destination accumulates mutual information from all the relays, the transmission rate of the equivalent relay is the sum over the transmission rates of the M relays. Therefore, $\frac{1}{S_{\text{eq}}^{\text{RD}}} = \frac{1}{B} \sum_{i=1}^{M} \log_e(1 + \lambda_i)$ (without provisioning for time out). Using variable transformations and characteristic functions ([6]), and accounting for time out, yields

$$p_{S_{\text{nd,eq}}^{\text{RD}}}(x) = \frac{B}{2\pi(1 - P_{\text{out}}^{\text{RD}})x^2} \int_{-\infty}^{\infty} \rho_{\text{RD}}^{-Mj\omega} \exp\left(\frac{j\omega B}{x}\right) \times \Gamma^M\left(1 - j\omega, \frac{1}{\rho_{\text{RD}}}\right) d\omega, \quad 0 \le x < t_{\text{out}}, \quad (27)$$

where $\Gamma(.,.)$ is the Incomplete Gamma function [16]. Further analytical simplification seems difficult. $P_{\text{out}}^{\text{RD}}$, $\mathbf{E}\left[S_{\text{nd,eq}}^{\text{RD}}\right]$, and $\mathbf{E}\left[(S_{\text{nd,eq}}^{\text{RD}})^2\right]$ are computed numerically from (27).

The queue at the equivalent relay is also a GI/GI/1 queue. Using the above results, we get the following stability result.

Theorem *3:* The relay queues for the strong inter-relay link scenario are stable if and only if

$$t_{\text{out}} \frac{P_{\text{out}}^{\text{RD}}}{1 - P_{\text{out}}^{\text{RD}}} + \mathbf{E}\left[S_{\text{nd,eq}}^{\text{RD}}\right] < t_{\text{out}} \frac{P_{\text{out}}^{\text{SR}}}{1 - P_{\text{out}}^{\text{SR}}} + \frac{1}{1 - P_{\text{out}}^{\text{SR}}} \frac{Bn}{\rho_{\text{SR}}} \sum_{i=0}^{M-1} (-1)^i \binom{n-1}{i} \psi_1\left(\frac{\rho_{\text{SR}}}{i+1}, \frac{B}{t_{\text{out}}}\right). \quad (28)$$

Finally, the end-to-end time and throughput can be calculated by substituting the above results in (2) and (18), respectively.

IV. Simulation Results

We now simulate the queued cooperative wireless networks (using 10^7 messages), and verify the accuracy of our analysis. The simulations assume a system bandwidth of 1 MHz, 4096 bits per message, a SR link SNR of $\rho_{\text{SR}} = 10$ dB, and $t_{\text{out}} = 10$ msec. The number of relays is $M = 3$, unless mentioned otherwise. In all figures, lines are used to plot the analytical results and markers ($\diamond$,$\square$, etc.) plot the simulation results.

Figure 2 shows both the throughput and the stability region for the queued relay selection scheme as a function of the RD link SNR for different numbers of relays. Both the throughput and stability region increase as the number of relays increases and as the RD link SNR improves. For example, relay selection is stable for $\rho_{\text{RD}} \ge 9.4$ for $M = 1$ and for $\rho_{\text{RD}} \ge 4.0$ for $M = 3$. For comparison sake, the stability regions of the traditional schemes and strong relay link cases are also shown for $M = 3$. Since the traditional schemes load the relays less, their stability region is larger ($\rho_{\text{RD}} \ge 1.0$), while the strong relay link case is stable only for $\rho_{\text{RD}} \ge 2.6$. Notice that the simulation and analytical results match well. While the throughput is greater for an unstable system (since the relay destination channel becomes the bottleneck), the delays become unbounded, which is not a desirable operating point for the system. The throughput of the conventional unbuffered case (not shown in the figure), in which the source transmits the next message only after receiving an acknowledgment from the destination, turns out to be considerably lower. For example, for relay selection with $M = 3$ relays, buffering improves throughput by 192% at $\rho_{\text{RD}} = 10$ dB and by 143% at $\rho_{\text{RD}} = 20$ dB.

Increasing the number of relays is beneficial not only because it increases spatial diversity and speeds up the source transmission times, but also because it reduces the load on each relay's queue and decreases queuing delays. The latter is seen clearly in Fig. 3, which plots the end-to-end time for the traditional queued relay networks. As M increases, the load at each relay queue decreases since the SR link throughput does not increase.

The end-to-end time for the various schemes is compared in Fig. 4 as a function of the RD link SNR, ρ_{RD}. For the same ρ_{RD}, relay selection has the highest end-to-end time followed by probabilistic selection and round-robin. This is because of the higher SR link throughput achieved by relay selection, which consequently loads the relay queues more compared to the traditional models. Notice also that the end-to-end delay of the round-robin scheme is always less than that for probabilistic selection. This is because the variability of the message arrival times at a relay's queue is lower for round-robin. The strong inter-relay case delivers the lowest end-to-end time. Again, the simulation and analytical results match well for all schemes.

A more fair comparison of the various schemes is achieved by comparing their throughputs for the same end-to-end time, as is done in Fig. 5. We now see that relay selection, which exploits spatial diversity, achieves a much higher throughput than the traditional schemes for the same end-to-end delay.

V. Conclusions

We analyzed how rateless codes can be used to transmit information through a two-hop cooperative wireless network, in which the relays have queues and the wireless links between

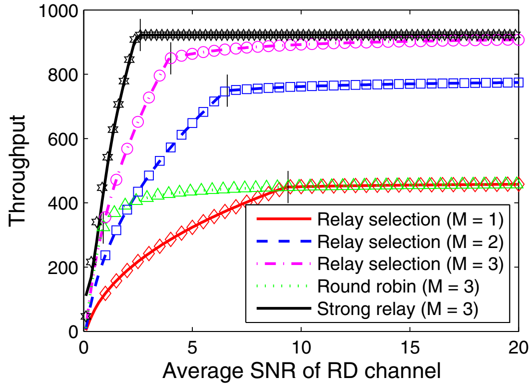

Fig. 2. Throughput and stability regions of various schemes. The short vertical marks indicate the stability region boundary.

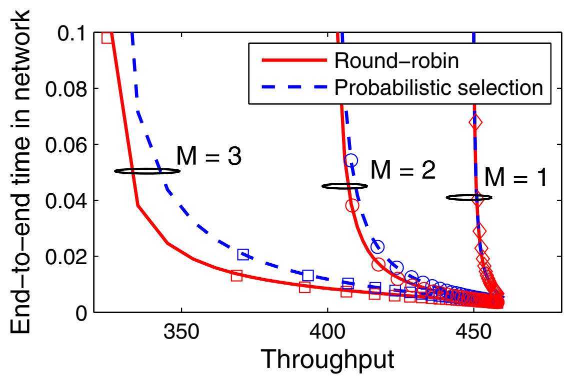

Fig. 3. Mean end-to-end message transit time in the network vs. throughput for traditional queued wireless networks for different numbers of relays.

them undergo Rayleigh fading. We saw that relay cooperation reduces not only transmission times (by exploiting spatial diversity), but also the end-to-end transit times (by providing load balancing). While the traditional queued relaying models of probabilistic selection and round-robin exploit only the latter aspect, relay selection does better by exploiting both the above aspects. However, we saw that relay selection has a smaller stability region than the traditional schemes.

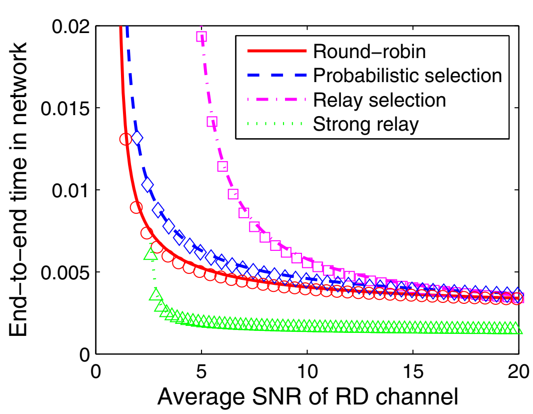

Fig. 4. Comparison of end-to-end message transit time in the network for $M = 3$ relays for different queued relaying schemes.

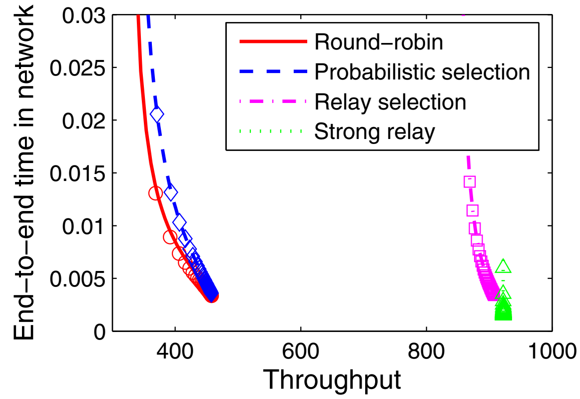

Fig. 5. Comparison of end-to-end message transit time in the network vs. throughput for $M = 3$ relays.

The strong inter-relay link scenario, in which strong inter-relay links enable all the relays to cooperate in transmitting messages to the destination, outperforms all the other schemes. This is because all the relays help in servicing every packet in the network, which is not the case with relay selection or the traditional schemes. Queuing messages at the relays led to a significant improvement in the system throughput. An interesting avenue for future research is developing transmission policies at the source that improve the stability of relay selection without reducing its throughput.

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