Distributed Transmission of Functions of Correlated Sources over a Fading Multiple Access Channel

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***Abstract*— In this paper we address the problem of distributed transmission of functions of correlated sources over a fast fading multiple access channel (MAC). This is a basic building block in a hierarchical sensor network used in estimating a random field where the cluster head is interested only in estimating a function of the observations. The observations are transmitted to the cluster head through a fast fading MAC. We provide sufficient conditions for lossy transmission when the encoders and decoders are provided with partial information about the channel state. Furthermore signal side information maybe available at the encoders and the decoder. Various previous studies are shown as special cases. Efficient joint-source channel coding schemes are discussed for transmission of discrete and continuous alphabet sources to recover function values.**

Keywords: Joint source-channel coding, Fading MAC, Power allocation, Partial channel state information, Graph coloring, Correlated sources.

I. Introduction and Survey

Sensor networks are characterized by inexpensive sensing nodes with limited battery power and storage and hence limited computing and communication capabilities. These networks may often be deployed for monitoring a random field. Due to the spatial proximity of the nodes, sensor observations are correlated.

The nodes in a sensor network may be arranged in a hierarchical fashion where neighboring nodes first transmit to a cluster head and then the cluster heads transmit to the fusion center, forming a network of multiple access channels (MACs). One often needs to transmit the observations to the cluster head through wireless links which experience multipath fading. The encoders and decoder often have only a partial information about the channel state as it has to be learnt. Also the cluster heads often need to send only a function of the sources to the fusion center.

In such a set up the sensor nodes can compress the data sent to the cluster head exploiting the correlation in the data and also the structure of the function to be computed at the cluster head. Depending upon the function, exploiting the structure of the function can substantially reduce the data rate for transmission. Since source-channel separation may not hold in this case even for independent sources ([15]), one needs joint source-channel codes to transmit over the MAC. We provide joint source-channel codes and optimal power allocation policies that can be used for transmission of functions over a fading multiple access channel where only partial information about the channel state is available at the encoders and the decoder.

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In the following we survey the related literature. The seminal paper, Cover, El Gamal and Salehi [2] provided sufficient conditions for transmitting losslessly correlated observations over a MAC. The results of [2] have been extended in [20] to the system with continuous alphabets, lossy transmission and side information.

The multi-access fading channels with independent inputs were considered in the excellent survey [1]. They show that unlike in the single user case, in the multi-access realm the optimal power control yields a substantial gain in capacity. The optimal power allocation strategy for the symmetric case is to allow the user with the best channel to transmit at a time (Random TDMA) ([10]). The instantaneous power allocated to a user is by the well known 'water-filling' algorithm in time.

Joint source channel coding for correlated data on a fading MAC with perfect CSIT and perfect CSIR is studied in [18]. Various power allocation schemes are studied and it is shown that the Random TDMA is suboptimal for transmission of correlated sources. Optimal power allocation strategies to minimize the sum distortion for transmission of discrete and Gaussian sources over a GMAC are given.

Korner and Marton [11] found the rate region for sending binary sum of two correlated uniform binary sources in a distributed manner. They show that $R_1 > H(f(U_1, U_2))$ and $R_2 > H(f(U_1, U_2))$ is achievable where R_1 and R_2 are the rates of transmission, (U_1, U_2) are the sources and $f(U_1, U_2) = U_1 \oplus U_2$, $\oplus$ denoting exclusive-OR function. Their proof relies on linear codes as opposed to the more prevalent random codes. Yamamato [24] found the rate-distortion function for sending a source X to a decoder that has Y as side-information and must estimate $f(X, Y)$ to a given distortion D. Orlitsky and Roche [16] show that the required rate for lossless transmission for the above problem is $H_G(X|Y)$ where $H_G(X|Y)$ denotes the conditional entropy of the characteristic graph. Feng, Effros, and Savari [8] address this problem with noisy observations.

Reference [13] uses the Korner-Marton scheme for finding achievable rate region for distributed source coding where the decoder is interested in lossy reconstruction of an arbitrary function. Linear codes were also used in [15] for joint source-channel codes that are optimal for a class of functions and channels. Exact rate region is obtained for a linear function over a finite field transmitted over concatenation of a linear MAC with a symmetric discrete memoryless channel.

Vishal et al. [5] provide a technique to separate out the functional coding and correlation coding by first coloring the graph and then applying Slepian-Wolf coding.

Further references on function coding problem can be found in [19] where joint source-channel coding of functions over a MAC using graph coloring is addressed.

This paper makes the following contributions. We provide joint source-channel coding schemes for computation of general functions over a general fading MAC under the assumption of partial CSI at the encoders and the decoder. The sources can be correlated and the source/channel alphabets can be discrete or continuous. There may be source side information available at the encoders and the decoder. The transmission can be lossy or lossless. We show that an efficient coding scheme involves exploiting the structure of the function, joint source-channel coding and an optimal power allocation policy. The existing literature on function transmission either studies only source coding or discrete sources over a discrete alphabet MAC. Also it mostly considers simple or specific functions. We show previous results as special cases of our results. The emphasis is on developing general techniques.

The rest of the paper is organized as follows. Section II gives sufficient conditions for lossy transmission of functions over a fading MAC. Special cases are given in Section III and Section IV considers joint source-channel coding using graph coloring. Transmission of functions over a fading GMAC with partial CSIT and CSIR is given in Section V. Section VI gives an example to motivate joint source-channel codes using graph coloring. Section VII considers continuous alphabet sources and/or channels. Section VIII concludes the paper. The proof of Theorem 1 is given in the Appendix.

II. Transmission of functions of sources over a fading MAC

We consider the transmission of a function of memoryless dependent sources, through a memoryless fading multiple access channel in the presence of source side information. The sources and/or the channel input/output alphabets can be discrete or continuous. The encoders and the decoder have partial information about the channel state.

We consider two sources (U_1, U_2) and source side information random variables Z_1, Z_2, Z with a known joint distribution $F(u_1, u_2, z_1, z_2, z)$. Side information Z_i is available to encoder i, $i = 1, 2$ and the decoder has side information Z. The random vector sequence $\{(U_{1n}, U_{2n}, Z_{1n}, Z_{2n}, Z_n), n \geq 1\}$ formed from the source outputs and the side information is iid in time with distribution F. The sources transmit their code words X_i's to a single decoder through a memoryless, flat, fast fading multiple access channel. Let (H_{1n}, H_{2n}) be the fade state at time n, $n \geq 1$. We assume $\{(H_{1n}, H_{2n}),\ n \geq 1\}$ to be an iid sequence, although (H_{1n}, H_{2n}) can be dependent and can be discrete or continuous valued. Similarly $(\hat{H}_{1n}, \hat{H}_{2n})$, $(\tilde{H}_{1n}, \tilde{H}_{2n})$ denote the channel state information available at the transmitters and the receiver respectively. $(H_{1n}, H_{2n}, \hat{H}_{1n}, \hat{H}_{2n}, \tilde{H}_{1n}, \tilde{H}_{2n})$ are iid in time. The channel output Y has distribution $p(y|x_1, x_2, h_1, h_2)$ if x_1 and x_2 are transmitted at that time and the channel is in the fade state (h_1, h_2). The decoder uses the channel output and its side information to estimate a function $G = f(U_1, U_2)$ of sensor observations as $\hat{G}$. It is of interest to find encoders and a decoder such that $\{U_{1n}, U_{2n}, n \geq 1\}$ can be transmitted over the given MAC with $E[d(G, \hat{G})] \leq D$ where d is a non-negative distortion measure and D is the given distortion constraint. We will assume that $d(a, a') = 0$ if and only if $a = a'$. We also assume that either d is upper bounded by $d_{max} < \infty$ or there exists v^* such that $E[d(f(U_1, U_2), v^*))] < \infty$. Two common distortion measures are $d(x_1, x_2) = (x_1 - x_2)^2$ and Hamming distance.

Due to correlated sources, source channel separation does not hold in this case.

We will denote $U_{ij}, j = 1, 2, ..., n$ by $U_i^n, i = 1, 2$. Also x will denote a realization of a random variable X.

$Definition$: The source (U_1^n, U_2^n) can be transmitted over the multiple access channel to recover the function $f(U_1, U_2)$ with distortion D if for any $\epsilon > 0$ there is an n_0 such that for all $n > n_0$ there exist encoders $f_{E,i}^n : \mathcal{U}_i^n \times \mathcal{Z}_i^n \times \hat{\mathcal{H}}_1^n \times \hat{\mathcal{H}}_2^n \rightarrow \mathcal{X}_i^n,\ i = 1, 2$ and a decoder $f_D^n : \mathcal{Y}^n \times \mathcal{Z}^n \times \tilde{\mathcal{H}}_1^n \times \tilde{\mathcal{H}}_2^n \rightarrow \hat{\mathcal{G}}^n$ such that $\frac{1}{n} E\left[\sum_{j=1}^n d(f(U_{1j}, U_{2j}), \hat{G}_j)\right] \leq D + \epsilon$ where $\hat{G}^n = f_D(Y^n, Z^n, \tilde{H}_1^n, \tilde{H}_2^n)$ and $\mathcal{U}_i,\ \mathcal{Z}_i,\ \mathcal{Z},\ \mathcal{X}_i,\ \mathcal{Y},\ \hat{\mathcal{G}},\ \mathcal{H}_i,\ \hat{\mathcal{H}}_i,\ \tilde{\mathcal{H}}_i$ are the sets in which $U_i,\ Z_i,\ Z,\ X_i,\ Y,\ \hat{G},\ H_i,\ \hat{H}_i,\ \tilde{H}_i$ take values.

We denote the joint distribution of (U_1, U_2) by $p(u_1, u_2)$. $X \leftrightarrow Y \leftrightarrow Z$ will indicate that $\{X, Y, Z\}$ form a Markov chain.

Now we state the main theorem.

Theorem 1: A function f of the sources (U_1, U_2) can be transmitted over the fading multiple access channel with distortion D if there exist random variables (W_1, W_2, X_1, X_2) such that

(1)
$$p(u_1, u_2, z_1, z_2, z, h_1, h_2, \hat{h}_1, \hat{h}_2, \tilde{h}_1, \tilde{h}_2, w_1, w_2, x_1, x_2, y) = p(u_1, u_2, z_1, z_2, z)\, p(h_1, h_2, \hat{h}_1, \hat{h}_2, \tilde{h}_1, \tilde{h}_2)\, p(w_1|u_1, z_1)\, p(w_2|u_2, z_2)\, p(x_1|w_1, \hat{h}_1, \hat{h}_2)\, p(x_2|w_2, \hat{h}_1, \hat{h}_2)\, p(y|x_1, x_2, h_1, h_2)$$

and

(2) there exists a function $f_D : \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{Z} \times \tilde{\mathcal{H}}_1 \times \tilde{\mathcal{H}}_2 \rightarrow \hat{\mathcal{G}}$ such that $E[d(G, \hat{G})] \leq D$, where $G = f(U_1, U_2),\ \hat{G} = f_D(W_1, W_2, Z, \tilde{H}_1, \tilde{H}_2)$ and the constraints

$$\begin{aligned} I(U_1, Z_1; W_1|W_2, Z) &< I(X_1; Y|X_2, W_2, Z, \tilde{H}_1, \tilde{H}_2), \\ I(U_2, Z_2; W_2|W_1, Z) &< I(X_2; Y|X_1, W_1, Z, \tilde{H}_1, \tilde{H}_2), \quad (1) \\ I(U_1, U_2, Z_1, Z_2; W_1, W_2|Z) &< I(X_1, X_2; Y|Z, \tilde{H}_1, \tilde{H}_2) \end{aligned}$$

are satisfied where $\mathcal{W}_i$ are the sets in which W_i take values.

If the channel input alphabets are continuous valued then the $X_i s$ should also satisfy given power constraints $E[X_i^2] \leq \overline{P}_i,\ i = 1, 2$.

Proof of Theorem 1 is given in the Appendix. Extension of this result to multiuser case is available.

In the proof of Theorem 1 the encoding scheme involves finding (W_1^n, W_2^n) from the sources (U_1^n, U_2^n) and the side information Z_1^n, Z_2^n by exploiting the structure of the function and the distortion permitted, followed by a correlation preserving mapping to the channel codewords (X_1^n, X_2^n) depending on the channel state. The correlation required should be such that the inequalities in (1) are satisfied. The decoding approach involves first decoding (W_1^n, W_2^n) and then obtaining the estimate $\hat{G}^n$ as a function of (W_1^n, W_2^n), the decoder side information Z^n and CSIR $(\tilde{H}_1^n, \tilde{H}_2^n)$.

One of the problems in applying Theorem 1 is that the auxiliary random variables W's are not explicit enough. We propose good joint source-channel coding schemes for obtaining W's in the subsequent sections.

III. Special Cases

In the following we show that our result contains several previous studies as special cases.

A. Lossy transmission of correlated sources with perfect CSIT, perfect CSIR

Take $(\hat{H}_1, \hat{H}_2) = (\tilde{H}_1, \tilde{H}_2) = (H_1, H_2)$ and $f(U_1, U_2) = (U_1, U_2)$ then we recover the conditions given in [18].

B. Lossless transmission of independent sources with partial CSIT, perfect/no CSIR

Take (U_1, U_2) as discrete valued and $(\tilde{H}_1, \tilde{H}_2) = (H_1, H_2)$. Also, $\mathbf{H} = (H_1, H_2)$, $U_1 \perp U_2$ (i.e., U_1 is independent of U_2). We also take $f(U_1, U_2) = (U_1, U_2)$ and $(W_1, W_2) = (U_1, U_2)$. Then we need to code at rate R_1 and R_2 satisfying

$$\begin{aligned} H(U_1) < R_1 &< I(X_1; Y|X_2, \mathbf{H}), \\ H(U_2) < R_2 &< I(X_2; Y|X_1, \mathbf{H}), \\ H(U_1 U_2) < R_1 + R_2 &< I(X_1, X_2; Y|\mathbf{H}). \end{aligned}$$

In the above equations X_1 and X_2 are generated using $(\hat{H}_1, \hat{H}_2)$ and are independent of (U_1, U_2).

These are the conditions given in [9].

If we take $(\tilde{H}_1, \tilde{H}_2) = (1, 1)$ (i.e., no CSIR) we obtain expressions with $\mathbf{H}$ removed from right side in the above inequality. This recovers the result in [21].

C. Lossy coding of functions with side information

Choose $Y = (X_1, X_2)$ in (1) and also $X_i = W_i,\ i = 1, 2$. Also choose $(\hat{H}_1, \hat{H}_2) = (\tilde{H}_1, \tilde{H}_2) = (H_1, H_2) = constant$ Then the R.H.S of (1) evaluate to $H(W_1|W_2), H(W_2|W_1)$ and $H(W_1, W_2)$ respectively. Thus we get a rate region

$$\begin{aligned} R_1 &> I(U_1, Z_1; W_1|W_2, Z), \\ R_2 &> I(U_2, Z_2; W_2|W_1, Z) \\ R_1 + R_2 &> I(U_1, U_2, Z_1, Z_2; W_1, W_2|Z). \end{aligned}$$

If we take $(Z_1, Z_2) \perp (U_1, U_2, Z)$ and $U_2 = W_2 = constant$, then $R_1 > I(U_1; W_1|Z)$ and we have a decoder such that from W_1 and Z it can estimate $f(U_1, Z)$ within distortion D. This recovers the result in [24].

D. Lossy coding of functions with side information and remote sources

The results of [8] can be recovered by applying the theorem as above to $\tilde{U}_1$ and $\tilde{Z}$ where $\tilde{U}_1$ and $\tilde{Z}$ are noisy versions of U_1 and Z. Similar to the above example we need $R_1 > I(\tilde{U}_1; W_1|\tilde{Z})$. We choose the distortion measure to allow dependence of the distortion measure on Z. So from the theorem we need $E[\tilde{d}(\tilde{U}_1, \tilde{Z}, g(W, \tilde{Z}))] \leq D$ where $\tilde{d}(\tilde{U}_1, \tilde{Z}, g(W, \tilde{Z}) = \sum_{(u_1 \times z) \in \mathcal{U}_1 \times \mathcal{Z}} p(u_1, z|\tilde{u}_1, \tilde{z}) d(f(u_1, z), g(w_1, \tilde{z}), z)$.

IV. Joint source-channel coding for discrete sources and channels

In this Section we extend the graph coloring technique given in [6] to joint source-channel coding of functions of discrete sources over a fast fading MAC. The channel alphabets can be discrete or continuous (in particular the channel can be a GMAC). For definition of graph related terminologies like *characteristic graph*, *conditional graph entropy*, *chromatic entropy*, *OR-product graph*, *conditional chromatic entropy* etc refer to [19], [22].

For large n we can color most of the OR-product graph G^n (the component of the graph G^n left uncolored has low probability) and send the colors to achieve an optimal coding scheme. Thus the graph coloring scheme can decouple the functional coding from the correlation coding [5].

In [6] the achievability results are shown for the distributed functional source coding problem provided the joint distribution satisfies a zigzag condition, i.e., $p(x_1, y_1) > 0$ and $p(x_2, y_2) > 0$ imply either $p(x_1, y_2) > 0$ or $p(x_2, y_1) > 0$. The achievable rate region is obtained by first coloring the OR-product graph and then compressing the colors through a Slepian-Wolf coding scheme.

Our coding scheme also relies upon coloring the graph and thus separating functional source coding and the correlation coding. We assume that the zigzag condition holds. If we allow distortion in evaluating the function we can color a subgraph of the original graph [4] using a choice of colors that achieve the minimum chromatic entropy. Once we color the graphs at both encoders, the correlated colors are sent over the fading MAC to the decoder using good joint source-channel coding schemes. The channel state information (CSI) available at the encoders are used for choosing a suitable codebook with an efficient power allocation policy and the CSI available at the decoder helps in decoding. The joint source-channel codes preserve the correlation in the colors and thus help in combating channel noise. The effectiveness of joint source-channel codes is shown in [17] and [20]. Using joint source-channel coding schemes the colors are losslessly recovered at the decoder and the zigzag condition ensures that the function can be recovered within a given distortion D. We will demonstrate through examples that this

$$
\begin{aligned}
I(U_1;W_1|W_2) &< 0.5E\left[\log\left(1+\frac{|H_1|^2P_1(\hat{H}_1,\hat{H}_2)(1-\tilde{\rho}^2)}{{\sigma_N}^2}\right)\right],\\
I(U_2;W_2|W_1) &< 0.5E\left[\log\left(1+\frac{|H_2|^2P_2(\hat{H}_1,\hat{H}_2)(1-\tilde{\rho}^2)}{{\sigma_N}^2}\right)\right],\\
I(U_1,U_2;W_1,W_2) &< 0.5E\left[\log\left(1+\frac{|H_1|^2P_1(\hat{H}_1,\hat{H}_2)+|H_2|^2P_2(\hat{H}_1,\hat{H}_2)+2|H_1||H_2|\tilde{\rho}\sqrt{P_1(\hat{H}_1,\hat{H}_2)P_2(\hat{H}_1,\hat{H}_2)}}{{\sigma_N}^2}\right)\right].
\end{aligned}
\tag{2}
$$

joint source-channel coding scheme out-performs the scheme with Slepian-Wolf coding on colored outputs.

V. Fading Gaussian MAC with partial CSIT, perfect CSIR

In this section we consider transmission of functions through a Gaussian MAC (GMAC). Initially we do not consider the side information (Z_1, Z_2, Z). In a fading GMAC the channel output Y_n at time n is given by $Y_n = H_{1n}X_{1n} + H_{2n}X_{2n} + N_n$ where X_{1n} and X_{2n} are the channel inputs at time n and $\{N_n\}$ is iid with a Gaussian distribution and is independent of X_{1n} and X_{2n}. Also, $E[N_n] = 0$ and $var(N_n) = \sigma_N^2$. H_{1n} and H_{2n} are the fade states of the channel at time n. The power constraints on the channel inputs are $E[X_i^2] \leq \overline{P}_i,\ i = 1, 2$. The distortion measure will be Mean Square Error (MSE). Let $\tilde{\rho}$ be the correlation between the channel inputs X_1, X_2.

For this GMAC, following the experience in [17] we relax the first two inequalities in (1) to make them more explicit. These are then used to obtain efficient signaling schemes to satisfy (1). For this the right hand side (RHS) of the first two inequalities in (1) are replaced by upper bounds $I(X_1; Y|X_2, H_1, H_2)$ and $I(X_2; Y|X_1, H_1, H_2)$ respectively. It is shown in [17] that these upper bounds are quite tight whenever these two inequalities are active (generally it is the third inequality which is tight). Also, it is shown in [17] that for a given (h_1, h_2), these upper bounds and the RHS of the third inequality in (1) are maximized by choosing (X_1, X_2) to be zero mean, jointly Gaussian r.v.s with $E[X_i^2] = P_i(\hat{h}_1, \hat{h}_2)$ where $P_i(\hat{h}_1, \hat{h}_2)$ are appropriately chosen. If such (X_1, X_2) have correlation $\tilde{\rho}$ these three bounds provide (2).

The expectation in (2) is over the joint fade state $(h_1, h_2, \hat{h}_1, \hat{h}_2)$. We also need to choose the power control policies $P_i(\hat{h}_1, \hat{h}_2)$ such that the average power constraints

$$E[P_i(\hat{H}_1, \hat{H}_2)] \leq \overline{P}_i,\ i = 1, 2,$$

are satisfied. This motivates us to consider Gaussian coding schemes.

An advantage of (2) is that we will be able to obtain explicit source-channel coding schemes to satisfy (2). These may be difficult to identify from (1) itself. Once we have obtained these coding schemes we can verify the sufficient conditions (1) themselves. If satisfied these will ensure that the coding schemes can ensure transmission with given distortions. If not, one can change $\tilde{\rho}$ to finally satisfy (1).

We also consider the following power allocation policies. We provide an optimal power allocation policy such that the RHS in the third inequality of (2) is maximized and the other conditions are satisfied. This is compared with Random Time division multiple access (RTDMA) and uniform power allocation (UPA). In RTDMA only the user having the best channel conditions transmits. If the best channel state is obtained by more than one user, then one of these users is selected with equal probability. Once a user is chosen the power allocation is by water-filling over the channel states of that user conditioned on the user being selected. This policy maximizes the sum of channel rates for independent sources and symmetric channel statistics and power constraints. In UPA both the users transmit all the time at powers P_1 and P_2.

VI. Example

Let (U_1, U_2) take values in the set $\{1, 2, 3\}$ and be jointly distributed with $p(u_1, u_2) = 1/6$ for all $u_1 \neq u_2$. The function to be computed at the receiver is $f(u_1, u_2) = 1$ if $u_1 > u_2$ and 0 otherwise. The zig-zag condition holds for this case. Let the sources be transmitted over a GMAC. The power constraints on the channel inputs (X_1, X_2) are $(3.5, 3.5)$. The channel noise is zero mean with unit variance. The sum capacity of this channel with independent (X_1, X_2) is 1.5 bits.

We consider transmitting the sources losslessy. The characteristic graph defined on the set $\{1, 2, 3\}$ at encoder 1 with respect to f and U_2 will have a single edge between 1 and 3. Similarly for encoder 2. Now we consider coloring of the graph. Let $C_1(1) = C_1(2) = 0$ and $C_1(3) = 1$ at encoder 1. Similarly define C_2 for the second encoder. The coloring takes into account the structure of the function. We compare various cases in Table I.

TABLE I
Comparison of various schemes

Condition	Sum-rate(bits)
Without function structure and correlation	3.16
Without function structure but with correlation	2.58
With function structure but without correlation	1.84
With function structue and correlation using S-W	1.58
Side information $Z = \|U_1 - U_2\|$ at decoder, colors compressed using S-W	1.32

Next we consider a joint source-channel code where the colors are mapped to correlated channel alphabets. Let the sources be mapped to channel codewords with correlation

$\tilde{\rho} = 0.4$. Such correlation preserving mappings are discussed in [17]. Then the sum capacity improves to 1.72 bits and it supports lossless transmission of the function of sources with graph coloring. Thus we notice that we can exploit the correlation between the sources, the structure of the function and the structure of the channel to construct efficient joint source-channel coding schemes via graph coloring.

Now we assume that the MAC experiences fading. The fade states (h_1, h_2) take values in $(1.2, 0.6)$ with equal probability and are independent of each other. We consider both perfect and partial CSI. Partial CSIT is generated by a BSC of suitable cross over probability p. We consider Joint source-channel coding with channel codewords with correlation $\tilde{\rho} = 0.4$ as above. Various cases are tabluated in Table II.

TABLE II

FADING CHANNEL

Condition	Sum Capacity
RTDMA, Perfect CSIT, CSIR	1.53 bits
UPA, Perfect CSIT, CSIR	1.57 bits
Optimal scheme, Perfect CSIT, CSIR	1.65 bits
Optimal scheme, Perfect CSIR, Partial CSIT ($p = 0.2$)	1.6 bits
Optimal scheme, Perfect CSIR, Partial CSIT ($p = 0.4$)	1.57 bits

VII. JOINT SOURCE CHANNEL CODING FOR CONTINUOUS SOURCES AND FADING MAC

In this section we consider joint source-channel coding for continuous alphabet sources.

Let f be Lipschitz: there is an $\alpha > 0$ such that $|f(u_1, u_2) - f(u_1', u_2')| < \alpha d((u_1, u_2), (u_1', u_2'))$ for all (u_1, u_2), (u_1', u_2'). Then $E[|f(U_1, U_2) - f(\hat{U}_1, \hat{U}_2)|] < \alpha E[d((U_1, U_2), (\hat{U}_1, \hat{U}_2))]$ where $(\hat{U}_1, \hat{U}_2)$ is an estimate of (U_1, U_2) at the decoder. Thus for distortion within D in function estimation, if $E[d((U_1, U_2), (\hat{U}_1, \hat{U}_2))] < \delta$ where $\delta = D/\alpha$, our requirements on function estimation are satisfied. This motivates us to obtain the following schemes. Of course these schemes can be used in conjunction with other properties of f (instead of Lipschitz as mentioned above).

A. Coding of U_i, $i = 1, 2$

Scheme 1: Obtain (W_1^n, W_2^n) via vector quantization of (U_1^n, U_2^n) (with distortion $\leq \delta$). Transmit (W_1^n, W_2^n) losslessly via a joint source-channel coding scheme and optimal power allocation. Approximate $f(U_1, U_2)$ at the decoder via $f(W_1, W_2)$. This scheme exploits only the Lipschitz property of the function.

Scheme 2: Obtain (W_1^n, W_2^n) via vector quantization as in scheme 1. Apply graph coloring to (W_1^n, W_2^n) as in Section IV. These colors are further compressed via Slepian-Wolf coding. The compressed colors are sent over the MAC using independent channel code words and using RTDMA. RTDMA is optimal for independent channel code words. This scheme corresponds to the separation based (SB) scheme ([15]) for function coding.

Scheme 3: Obtain (W_1^n, W_2^n) and apply graph coloring as in scheme 2. The colors are sent over the MAC using a joint source-channel code and an optimal power allocation scheme and the functions are estimated from the colors.

We compare the performance of the three schemes via examples. In all our examples below we see that scheme 3 performs the best although sometimes scheme 1 and scheme 3 provide the same outcome. In the following we not only consider Lipschitz functions but also others to illustrate that these ideas have general applicability.

B. Difference of Gaussian sources over a Gaussian MAC

We consider correlated Gaussian sources (U_1, U_2) with mean zero, variance σ^2 and positive correlation ρ and are interested in estimating $U_1 - U_2$ at the decoder. Each of the channel inputs is power constrained to P and the sources are sent over a GMAC with receiver noise variance $= 1$. This function has been considered in [12] and [23].

We compare the performance of the above schemes along with a few other schemes natural for this setting. Some of these schemes have been already studied in literature. Of course as against these schemes the three schemes proposed above are applicable to general classes of functions, sources and channels. In the first part of this subsection we do not consider fading.

To put the performance of different coding schemes in a proper perspective, we also consider a lower bound on achievable distortion by considering a centralized encoder which has both the sources. The channel input $X = \alpha(U_1 - U_2)$ is power constrained to $2P$. Hence it is equivalent to a point to point channel given by $Y = X + V$ where V is independent Gaussian noise with zero mean and variance 1. The lower bound obtained via this system on the mean square distortion (MSE) of the function is given by

$$D_{cen} = \frac{2\sigma^2(1-\rho)}{1+2P}. \tag{3}$$

We also study a scheme where $X_1 = \sqrt{P}U_1$ and $X_2 = -\sqrt{P}U_2$. In this scheme the channel codewords are scaled versions of the source. Thus we call it Amplify and Forward (AF). At the decoder the difference of the sources are estimated from the channel output as $E[(U_1 - U_2)|Y]$. Then the MSE is

$$D_{AF} = \frac{2\sigma^2(1-\rho)}{1+2P(1-\rho)}. \tag{4}$$

From (3) and (4) we see that D_{AF} approaches D_{cen} at low ρ. A similar conclusion holds if f is any additively separable function. Of course this scheme exploits the Gaussian structure of the sources and the channel. One does not expect it to work well for general sources and channels.

Lattice coding schemes in [7] have been shown to improve upon the AF, discussed above for this system. We use the Lattice coding scheme provided in [23] designed specifically for this example.

Next we consider Scheme 2. For the specific case of difference of Gaussian sources over a GMAC, the characteristic graph formed from (W_1, W_2) and f is complete. Hence

$H_G(X) = H(X)$ and coloring the graph does not give any advantage. This will often happen for sources with density. The colors are compressed via Slepian-Wolf and are sent over the MAC using independent channel code words.

Next consider Scheme 1. In the example the quantized sources are mapped to Gaussian codewords, using the correlation preserving mapping provided in [14]. At the decoder the function is estimated from the colors. Because the graph is complete Scheme 3 is same as Scheme 1.

We compare the schemes discussed above in Fig. 1 for $\rho = 0.5$. Since Lattice codes need $P > 0.5$ for error free decoding, we compare the schemes only in this region. From the figure we see that Scheme 1 performs better than scheme 2 for all SNR and ρ. Also for lower ρ, AF is closer to the lower bound than the Lattice coding scheme. In this example, since the function was matched to the GMAC, AF and Lattice codes provide considerable gains. We will show by an example that this is not always true.

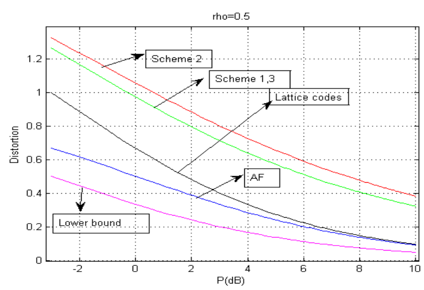

Fig. 1. Comparison of schemes for joint source-channel coding of functions, $\rho = 0.5$

Next we consider the GMAC with fast fading and only partial information about the channel state is available at the encoders. The fade states take values in $(1, 0.5)$. Like in the previous sections the partial CSIT is modeled as the output of a BSC with $p = 0.25$. The optimal power allocation policy chosen is such that it minimizes the distortion. We compare the performance of scheme 1, 2 and 3 with AF for transmitting the difference $U_1 - U_2$ for $\rho = 0.5$ in figure 2.

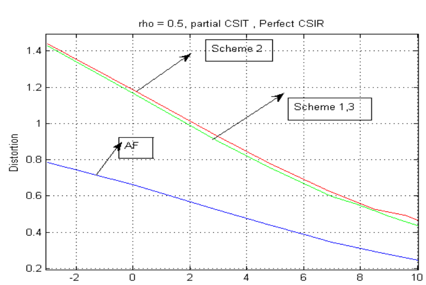

Fig. 2. Comparison of schemes for joint source-channel coding of functions over a fading GMAC, $\rho = 0.5$

From Figs 1 and 2 we see that the effect of fading and partial CSIT is to increase the distortion. Also, Scheme 2 performs closer to Schemes 1 and 3 in this case. This effect is due to the loss in efficiency of power allocation due to imperfect information about the channel state.

C. Binary function of Gaussian sources over a fading GMAC

Consider the sources to be the same as in the above example and the sources are to be transmitted over a Fading GMAC where perfect CSI is available at the encoders and the decoder. The fade states h_1, h_2 take values in $(1, 0.75)$. The power constraints on the channel inputs (X_1, X_2) are $(5, 5)$. The function to be computed is

$$f(min(U_1, U_2) > 0) = 1, \;\; f(min(U_1, U_2) \leq 0) = 0.$$

The natural $W_i's$ for this case are as follows. The W_i at encoder i is $= 1$, if $U_i > 0$ else $W_i = 0$. The (W_1, W_2) thus obtained are correlated. We color the $W_i's$ at each encoder. Since the characteristic graph is complete, coloring does not give any advantage. For $\rho = 0.75$ and $P = 5$, by Slepian- Wolf coding (W_1, W_2) can be compressed to $H(W_1, W_2) = 1.778\ bits$ (this scheme is similar to scheme 2). If we use RTDMA to transmit the sources then the sum-rate evaluates to 1.6386 bits and hence (W_1, W_2) cannot be reliably transmitted.

Next we send (W_1, W_2) using a joint-source channel coding scheme (similar to scheme 1 and 3). The correlation between (W_1, W_2) is 0.54 and we consider mappings of (W_1, W_2) to jointly Gaussian codewords with correlation $\tilde{\rho} = 0.3$. Then the channel supports sum rates till 1.7936 bits for the optimal power allocation scheme and hence the function can be losslessly computed. However if we use UPA instead of the optimal scheme the sum rate is 1.7726 bits and hence (W_1, W_2) cannot be transmitted reliably.

Next we assume that only partial CSIT is available. The partial CSIT is modeled as the output of a BSC with suitable crossover probability p as in the previous example. For $p = 0.1$ the sum rate is 1.7864 bits and hence this partial CSIT supports transmission. However for $p = 0.3$ sum-rate is 1.7762 and hence we are not able to transmit (W_1, W_2) over the GMAC losslessly.

If we consider AF or the Lattice coding scheme, the sources cannot be recovered losslessly at the decoder (since P is finite) and hence the function also cannot be losslessly computed with these schemes.

VIII. Conclusions

In this paper we provide sufficient conditions for lossy transmission of functions over a MAC which experiences fast fading. Efficient joint source-channel coding schemes are developed for transmission of discrete and continuous alphabet sources. Suitable power allocation policies are provided which support efficient transmission. It is shown that by exploiting the function structure substantial savings in transmission rate can be obtained. Although, a good coding scheme can depend upon the function, the source statistics and the channel, we identify a scheme which performs well for various classes of functions, sources and channels.

Appendix

We first provide the proof for discrete sources and channel alphabets and bounded distortion metric.

Proof: Fix $p(w_1|u_1,z_1)$, $p(w_2|u_2,z_2)$, $p(x_1|w_1,\hat{h}_1,\hat{h}_2)$, $p(x_2|w_2,\hat{h}_1,\hat{h}_2)$ as well as f_D satisfying constraints of the form $E[d(f(U_1,U_2),g(W_1,W_2,Z,\tilde{H}_1,\tilde{H}_2))] \leq D$, where g is a deterministic function.

Codebook generation: Let $R_i^{'} = I(U_i,Z_i;W_i)+\delta,\ i = 1,2$ for some $\delta > 0$. Generate $2^{nR_i^{'}}$ codewords of length n, sampled iid from the marginal distribution $p(w_i),\ i = 1,2$. For each w_i^n and $(\hat{h}_1^n,\hat{h}_2^n)$ independently generate sequence X_i^n according to $\prod_{j=1}^{n} p(x_{ij}|w_{ij},\hat{h}_{1j},\hat{h}_{2j}),\ i = 1,2$. Call these sequences $x_i^n(w_i^n,\hat{h}_1^n,\hat{h}_2^n), i \in 1,2$. Reveal the codebooks to the encoders and the decoder.

Encoding: For $i = 1,2$, given the source sequence $u_i^n,\ z_i^n$ and $\hat{h}_1^n,\hat{h}_2^n$, the i^{th} encoder looks for a codeword w_i^n such that $(u_i^n,z_i^n,w_i^n) \in T_\epsilon^n(U_i,Z_i,W_i)$ and then transmits $X_i^n(W_i^n,\hat{h}_1^n,\hat{h}_2^n)$ where $T_\epsilon^n(.)$ is the set of weakly ϵ-typical sequences ([3]) of length n. Although $\{w_{ik}, k \geq 1\},\ i = 1,2$, are generated as iid sequences, since w_i^n is chosen jointly typical with (u_i^n,z_i^n), for large n, $p(u_i^n,z_i^n,w_i^n)$ is close to that for a sequence where w_i^n would have been obtained with distribution $\prod_{j=1}^{n} p(w_{ij}|u_{ij},z_{ij}), i = 1,2$. Thus we can use the extended Markov Lemma ([18]) and hence $(u_1^n,z_1^n,w_1^n,w_2^n,z_2^n,u_2^n)$ is jointly typical.

Decoding: Upon receiving Y^n, for a given $(\tilde{h}_1^n,\tilde{h}_2^n)$ the decoder finds the unique (W_1^n,W_2^n) pair such that $(W_1^n,W_2^n,x_1^n(W_1^n,\overline{h}_1^n,\overline{h}_2^n),x_2^n(W_2^n,\overline{h}_1^n,\overline{h}_2^n),Y^n,Z^n) \in T_\epsilon^n$, where $\overline{h}_1^n,\overline{h}_2^n$ takes values in $\hat{\mathcal{H}}_1^n,\hat{\mathcal{H}}_2^n$. If it fails to find such a unique pair, the distortion incurred is d_{max}.

In the following we show that the probability of error for the above encoding, decoding scheme tends to zero as $n \rightarrow \infty$. By Markov Lemma, which holds for weakly typical sequences [18], $P\{(U_1^n,U_2^n,W_1^n(U_1^n),W_2^n(U_2^n),X_1^n(W_1^n,\hat{h}_1^n,\hat{h}_2^n), X_2^n(W_2^n,\hat{h}_1^n,\hat{h}_2^n),Y^n,Z^n) \in T_\epsilon^n\} \rightarrow 1$ as $n \rightarrow \infty$. The error can occur because of the following four events **E1**-**E4**. We show that $P(\mathbf{Ei}) \rightarrow 0$, for $i = 1,2,3,4$. For simplicity we take $\delta = \epsilon$.

E1 The encoders do not find the codewords. However from rate distortion theory ([3], P. 356) for $R_i^{'} > I(U_i,Z_i;W_i), i \in 1,2$, $\lim_{n\rightarrow\infty} P(\mathbf{E_1}) = 0$.

E2 The codewords are not jointly typical with Z^n. Probability of this event goes to zero from the extended Markov Lemma.

E3 There exists another codeword $\hat{w}_1^n$ such that $(\hat{w}_1^n,W_2^n,x_1^n(\hat{w}_1^n,\overline{h}_1^n,\overline{h}_2^n),x_2^n(W_2^n,\overline{h}_1^n,\overline{h}_2^n),Y^n,Z^n) \in T_\epsilon^n$. Define $\alpha \stackrel{\Delta}{=} (\hat{w}_1^n,W_2^n,x_1^n(\hat{w}_1^n,\overline{h}_1^n,\overline{h}_2^n),x_2^n(W_2^n,\overline{h}_1^n,\overline{h}_2^n),y^n,z^n)$. Then,

$$P(\mathbf{E3}) = \sum_{w_1^n} p(w_1^n) P\{\text{There is } \hat{w}_1^n \neq w_1^n : \alpha \in T_\epsilon^n\}$$
$$\leq \sum_{w_1^n} p(w_1^n) \sum_{\hat{w}_1^n \neq w_1^n : (\hat{w}_1^n, W_2^n) \in T_\epsilon^n} P\{\alpha \in T_\epsilon^n\}. \quad (5)$$

Denote $\{(x_1^n(.),x_2^n(.),y^n) : \alpha \in T_\epsilon^n\}$ by A. The probability term inside the inner summation in (5) is

$$Pr\{\alpha \in T_\epsilon^n\} \leq 2^{-nI(X_1;Y|X_2,W_2,\tilde{H}_1,\tilde{H}_2,Z)}$$

Then from (5)

$$\sum_{\hat{w}_1^n \neq w_1^n : (\hat{w}_1^n, w_2^n, z^n) \in T_\epsilon^n} Pr\{\alpha \in T_\epsilon^n\}$$
$$\leq \sum_{\hat{w}_1^n \neq w_1^n : (\hat{w}_1^n, w_2^n) \in T_\epsilon^n} 2^{-n\{I(X_1;Y|X_2,W_2,\tilde{H}_1,\tilde{H}_2,Z)-6\epsilon\}}$$
$$\leq 2^{n\{I(U_1,Z_1;W_1|W_2,Z)\}} 2^{-n\{I(X_1;Y|X_2,W_2,\tilde{H}_1,\tilde{H}_2,Z)-8\epsilon\}}. \quad (6)$$

Thus

$$P(\mathbf{E3}) \leq 2^{n\{I(U_1,Z_1;W_1|W_2,Z)-I(X_1;Y|X_2,W_2,\tilde{H}_1,\tilde{H}_2,Z)-9\epsilon\}} \sum_{w_1^n} p(w_1^n). \quad (7)$$

The RHS of (7) tends to zero if $I(U_1,Z_1;W_1|W_2,Z) < I(X_1;Y|X_2,W_2,\tilde{H}_1,\tilde{H}_2,Z)$.

Similarly, by symmetry of the problem we require

$$I(U_2,Z_2;W_2|W_1,Z) < I(X_2;Y|X_1,W_1,\tilde{H}_1,\tilde{H}_2,Z). \quad (8)$$

E4 There exist other codewords $\hat{w}_1^n$ and $\hat{w}_2^n$ such that $\alpha \stackrel{\Delta}{=} (\hat{w}_1^n,\hat{w}_2^n,x_1^n(\hat{w}_1^n,\overline{h}_1^n,\overline{h}_2^n),x_2^n(\hat{w}_2^n,\overline{h}_1^n,\overline{h}_2^n),y^n,z^n) \in T_\epsilon^n$.

Following the steps similar to that for $P(\mathbf{E3})$, $P(\mathbf{E4})$ approaches zero for large n if $I(U_1,U_2,Z_1,Z_2;W_1,W_2|Z) < I(X_1,X_2;Y|\tilde{H}_1,\tilde{H}_2,Z)$.

Thus as $n \rightarrow \infty$, with probability tending to 1, the decoder finds the correct sequence (W_1^n,W_2^n) which is jointly weakly ϵ-typical with $(U_1^n,U_2^n,Z_1^n,Z_2^n,Z^n)$.

The fact that (W_1^n,W_2^n) are weakly ϵ-typical with (U_1^n,U_2^n,Z^n) does not guarantee that $f_D^n(W_1^n,W_2^n,Z^n)$ will satisfy the distortion D. For this, one needs that (W_1^n,W_2^n) are distortion weakly ϵ- typical (in the sense that $d(f(u_1^n,u_2^n),f_d^n(w_1^n,w_2^n,z^n)) \leq D$ is also satisfied by the weakly ϵ-typical set) ([3]) with (U_1^n,U_2^n,Z^n). Let $T_{D,\epsilon}^n$ denote the set of distortion typical sequences ([3]). Then by strong law of large numbers $P(T_{D,\epsilon}^n|T_\epsilon^n) \rightarrow 1$ as $n \rightarrow \infty$. Thus the distortion constraints are also satisfied by (W_1^n,W_2^n) obtained above with a probability tending to 1 as $n \rightarrow \infty$. Therefore, if distortion measure d is bounded $\lim_{n\rightarrow\infty} E[d(f(U_1^n,U_2^n)),f_D(W_1^n,W_2^n,Z^n)] \leq D + \epsilon$.

Unbounded distortion measure

When the distortion measure d is unbounded then we proceed as follows. Since $E[d(f(U_1,U_2),v^*))] < \infty$, we can choose u_1^* and u_2^* such that $E[d(f(U_1,u_2^*),v^*))] < \infty$ and $E[d(f(u_1^*,U_2),v^*))] < \infty$. Take $M < \infty$ a conveniently large positive constant, larger than $max(E[d(f(U_1,u_2^*),v^*)],E[d(f(u_1^*,U_2),v^*)]) < \infty$.

We will encode U_1^n,U_2^n only if

$$\sum_{j=1}^{n} \frac{d(f(U_{1j},u_2^*),v^*)}{n} \leq M \ and$$
$$\sum_{j=1}^{n} \frac{d(f(u_1^*,U_{2j}),v^*)}{n} \leq M. \quad (9)$$

If this condition is not met then a specific codeword (say e) of length n is sent which is not used as a codeword in any of the codebooks generated above. If the receiver decodes either (or both) W_i^n as e then the estimated $\hat{G}^n$ is v^{*n}. If (9) holds then we encode and decode as before. Also if receiver knows that (9) holds then it will provide estimate $\hat{G}^n$ that satisfies (9). Since M is larger than $max(E[d(f(U_1, u_2^*), v^*)], E[d(f(u_1^*, U_2), v^*)])$, (9) will be satisfied with a large probability for all n large enough.

Define events

$$A_n = \{(9)\ holds\},$$
$$C_n = \{Receiver\ knows\ correctly\ if\ A_n\ holds\ or\ A_n^c\}$$

Then

$$E[d(G^n, \hat{G}^n)] = E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_n}] + E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_n^c}] + E[d(G^n, \hat{G}^n)\mathbf{1}_{A_n^cC_n}] + E[d(G^n, \hat{G}^n)\mathbf{1}_{A_n^cC_n^c}] \quad (10)$$

We will compute each quantity on the right side separately. From the arguments given above, we can show that $\lim_{n\to\infty} P(A_nC_n) = 1$.

Define E_n as the event that (W_1^n, W_2^n) is decoded with error. Then

$$E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_n}] = E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_nE_n}] + E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_nE_n^c}].$$

Since $P[A_nC_nE_n^c] \to 1$ and by assumption $E[d(G^n, \hat{G}^n)|E_n^c] \le D$, for any $\epsilon_1 > 0$,

$$E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_nE_n^c}] \le D + \epsilon_1$$

for all large n. Also when U_i^n satisfies (9) then knowing (9) $\hat{G}^n$ should be picked such that $d(G^n, \hat{G}^n) \le M_1$ for some $M_1 < \infty$. Thus,

$$\lim_{n\to\infty} E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_nE_n}] = 0.$$

When $A_nC_n^c$ holds then $\hat{G}^n = v^{*n}$. Therefore

$$E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_n^c}] = E[d(G^n, v^{*n})\mathbf{1}_{A_nC_n^c}].$$

Since $E[d(G^n, v^{*n})] < \infty$ and $P(A_nC_n^c) \to 0$, we obtain

$$\lim_{n\to\infty} E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_n^c}] = 0.$$

If $A_n^cC_n$ holds then $\hat{G}^n = v^{*n}$. Thus $P(A_n^cC_n) \to 0$ implies

$$E[d(G^n, \hat{G}^n)\mathbf{1}_{A_nC_n^c}] = E[d(G^n, v^{*n})\mathbf{1}_{A_nC_n^c}] \to 0\ as\ n \to \infty.$$

When $A_n^cC_n^c$ holds then $\hat{G}^n$ is estimated as some symbol satisfying (9). Now,

$$d(G^n, \hat{G}^n) \le d(G^n, v^{*n}) + d(v^{*n}, \hat{G}^n).$$

But $E[d(G^n, v^{*n})\mathbf{1}_{A_n^cC_n^c}] \to 0\ as\ n \to \infty$. Also $d(v^{*n}, \hat{G}^n)$ is bounded and hence

$$\lim_{n\to\infty} E[d(v^{*n}, \hat{G}^n)\mathbf{1}_{A_n^cC_n^c}] = 0.$$

Thus

$$\lim_{n\to\infty} E[G^n, \hat{G}^n)\mathbf{1}_{A_n^cC_n^c}] = 0.$$

The above results along with (10) show that for n large enough $E[d(G^n, \hat{G}^n] < D + \epsilon$.

The above proof also hold for continuous alphabet sources and channels. The Markov lemma and weak typical decoding, the devices used to prove the theorem continue to hold and the proof extends with $E[X_i^2] \le \overline{P}_i\ i = 1, 2$. ■

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