

# Amplify and Forward for Correlated Data Gathering over Hierarchical Sensor Networks

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**Abstract**—We address the problem of estimating a random field via a wireless sensor network. We use a Multiple Access Channel (MAC) as the basic building block for such a network. For Gaussian sources over Gaussian MACs, we show that Amplify and Forward scheme (AF) performs well in such sensor network scenarios where the battery power is at a premium. We then extend this result to the hierarchical network scenario and show that it can perform favourably to the Slepian-Wolf based source coding and independent channel coding scheme. Since AF is simple and scalable, a good performance makes it an attractive scheme.

**Keywords:** Multiple access channel, Amplify and Forward, Hierarchical network, Sensor network, Data gathering.

## I. INTRODUCTION AND SURVEY

Sensor nodes are often deployed for monitoring a random field. Due to spatial proximity, the observations at neighboring sensor nodes are correlated. This leads to the problem of data gathering from correlated sources. Also, the sensor network may be organized in a hierarchical way ([2], [17], [18], [23]) where there are usual sensor nodes and (possibly) more powerful cluster heads. Neighboring nodes can first transmit their data to a cluster head which can further compress the information before transmission to the fusion node (sink). The cluster heads may sense the random field in addition to relaying the data from other sensors. This can be viewed as a hierarchy of Multiple Access channels (MACs). We show that for a Gaussian hierarchical network, Amplify and Forward (AF) is an attractive and efficient transmission scheme.

In the following we survey the related literature. Cover, El Gamal and Salehi [4] provided sufficient conditions for transmitting losslessly discrete correlated observations over a discrete MAC. They also show that unlike for independent sources, the source-channel separation does not hold. These techniques were extended to more general models with discrete sources and channels and lossless transmission in [1]. Reference [22] extends the result in [4] and obtains sufficient conditions for lossy transmission of discrete/continuous correlated sources over a MAC with side information.

The sensor observations can be modeled as discrete or continuous sources. For continuous sources, Gaussian distribution is particularly useful. This for example can happen if the sensor nodes are sampling a Gaussian random field ([13], [15]). Another example is detection of change ([25]), a killer

application in sensor networks. In this application, one often detects change in the mean of the sensor observations with the observation noise being Gaussian. Thus, in this paper we focus on transmission of Gaussian sources over a hierarchical network.

For point to point transmission of Gaussian sources over an additive white Gaussian noise (AWGN) channel, it is well known that uncoded transmission (AF) is optimal [14]. In [16] transmission of two Gaussian sources over a Gaussian MAC (GMAC) is considered and it is shown that for the symmetric case the amplify and forward (AF) scheme is optimal below a certain SNR determined by source correlations. Performance of AF is compared in [21] with other coding schemes in the above set up. In [20] transmission of Gaussian sources over a Gaussian MAC with side information is studied. In [19] it is shown that AF performs close to the optimal scheme ([26]) for lossy transmission of correlated Gaussian sources over orthogonal Gaussian channels with and without side information. In [10] also optimality of uncoded transmission is shown in some related systems. Reference [12] shows that separation holds and uncoded transmission achieves capacity in a Gaussian relay network as the number of relays go to infinity. The authors in [11] consider uncoded transmission in a CEO like Gaussian sensor network and study its scaling behavior. It is also proved that the uncoded transmission is asymptotically optimal as the number of sensors increases. Thus we see that AF has been shown to be a promising scheme for transmission of Gaussian sources on Gaussian channels in many scenarios. Since it is also the simplest coding and decoding scheme, if it performs well in a certain scenario, it should be the preferred scheme. Thus, in this paper we study its performance for Gaussian sources over a hierarchical network of Gaussian channels.

AF is widely studied in the context of co-operative communication also (see, e.g., [8], [27]). The main difference there is that the data is not correlated.

In [18] trade off between the coding rate and reliability is considered for several different Slepian-Wolf based coding schemes for single-hop field gathering sensor networks. Our work considers multi-hop lossy joint source-channel coding as opposed to lossless source coding in [18].

The sensor reach back problem is considered in [3] where the sensors transmit their correlated observations through finite capacity links. It is shown that separation holds in such a

This work was partially supported by the DRDO-IISc program on Advanced Research in Mathematical Engineering.

system. Side information aware coding strategies for tree structured sensor networks are discussed in [9]. Modular and de-centralized architectures based on generalization of Wyner-Ziv source coding with decoder side information are proposed. A multi-hop network is studied in [7]. The optimal transmission structure is characterized for a data gathering scenario and the optimal rate allocation for Slepian-Wolf coding ([24]) is provided. This scheme requires complete network knowledge. The lossy version of the problem using high resolution quantization is given in [6].

Our setup considers MAC with side information as the basic building block of such hierarchical networks as compared to noiseless rate limited bit pipes as in [3], [9] and [7]. Hence, our set-up is a generalization of [3], [9] and [7] and can be specialized to the system there by considering orthogonal channels with suitable power constraints. Also we include spatial reuse in our model. AF has the advantage of being the simplest coding and decoding scheme and it is scalable.

This paper makes the following contributions. It studies components of a hierarchical network and proposes an Amplify and Forward scheme for the Gaussian hierarchical network. Even for a GMAC we provide interesting results which were not known before. The superiority of this scheme as compared to the Slepian-Wolf based approach is established for a hierarchical sensor network scenario. The effectiveness of AF is shown for the network components as well as for the network.

The paper is organized as follows. The network model is given in Section II. Section III considers transmissions from the data nodes on a GMAC and Section IV considers a relay node. Section V analyzes the hierarchical network. Section VI concludes the paper.

## II. HIERARCHICAL NETWORK- MODEL

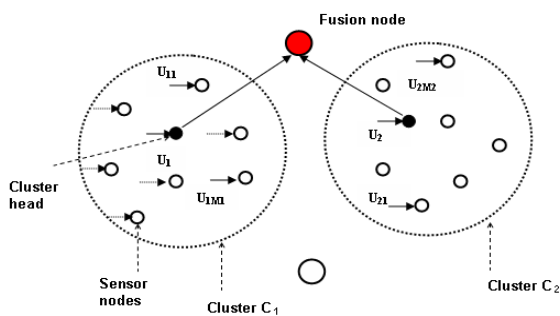


Fig. 1. Hierarchical network

In this section we describe our system. We consider a two level hierarchical network. The nodes are organized into clusters and each cluster has a cluster head. The nodes in each cluster transmit to their cluster head via a MAC. The cluster heads then transmit to the fusion node again via a MAC. It is assumed that the clusters do not interfere amongst themselves.

Let the network be divided into  $N$  clusters. The clusters are denoted by  $C_i$ ,  $i = 1, \dots, N$ . The nodes in cluster  $C_i$  are denoted as  $S_{ij}$ ,  $j = 1, \dots, M_i$ . The cluster head for  $C_i$  is denoted by  $H_i$ . The sensors  $S_{ij}$  in each cluster observe iid sequence of Gaussian r.v.s  $\{U_{ijn}, j = 1, \dots, M_i; i = 1, \dots, N; n \geq 1\}$  and cluster heads observe  $\{U_{in}, i = 1, \dots, N; n \geq 1\}$ . The r.v.s  $\{U_{ijn}, U_{in} j = 1, \dots, M_i, i = 1, \dots, N\}$  generated by different nodes at time  $n$  are correlated. We will denote by  $U_{ij}$  and  $U_i$  generic r.v.s from these sequences. The aim is to transmit each of the observations  $U_{ij}, U_i i = 1, \dots, N; j = 1, \dots, M_i$  to the sink such that the sum of the distortions incurred in reproducing the sensor observations at the sink is minimized. We will take mean square error as the distortion measure.

The transmission structure considered for this network is a GMAC. Transmissions from each sensor node within a cluster and from the cluster heads to the fusion node occur over a GMAC and are subject to average power constraints. Sensors transmit their observations over the GMAC taking into account the side information available at the cluster head. It is also assumed that a node cannot transmit and receive simultaneously (half duplex links).

Let  $Y_i$  denote the outputs of the GMAC for cluster  $C_i$ . Output  $Y_i$  is a superposition of the inputs and an independent Gaussian noise  $V_i$ . At the cluster head  $H_i$ ,  $Y_i$  and the corresponding node observations  $U_i$  are forwarded to the sink. Let  $Y$  denote the output at the sink node and let  $U$  be the observation sensed by the sink node. This structure permits spatial reuse and increases the network throughput and is also scalable to multiple levels of hierarchy.

In the following section we consider the network components with data nodes (the sensor nodes in each cluster) and relay nodes (cluster heads). We study their performance for transmissions via a GMAC using AF and compare to another recently studied efficient scheme.

## III. TRANSMISSIONS FROM DATA NODES

Consider the transmissions over a GMAC from data nodes. We study the performance of two coding schemes namely (a) AF and (b) Vector quantization followed by Slepian-Wolf coding and Time Division Multiple Access (TDMA) (SB-TDMA) for such transmissions. For symmetric conditions that we consider in the following, for SB, TDMA provides optimal performance for a GMAC. This is because under symmetric conditions TDMA achieves capacity of the GMAC [5]. SB-TDMA is a version of the scheme used in [7], [6] and [24] in our setup. We study the performance of these schemes for  $N$  sensor nodes transmitting through the GMAC. In this and next section there is only one MAC. Thus we denote  $U_{ij}$  by  $U_i$  and  $U_i$  by  $U_s$ .

### A. Amplify and Forward

In AF each observation  $U_i i = 1, \dots, N$  is scaled to the channel power constraint  $P$  and sent over the memoryless GMAC, i.e., node  $i$  transmits  $X_i = \alpha_i U_i$  where  $\alpha_i$  is chosen such that  $E[X_i^2] = P$ . The output of the GMAC is  $Y = X_1 + X_2 + \dots + X_N + V$  where  $V$  has a Gaussian

distribution and is independent of  $X_i$ ,  $i = 1, \dots, N$ . Also,  $E[V] = 0$  and  $\text{var}(V) = \sigma^2$ .  $(\hat{U}_1, \hat{U}_2, \dots, \hat{U}_N)$  are the estimates of  $U_1, U_2, \dots, U_N$  from  $Y$ . AF with two users and under symmetric conditions has been studied before. We study it for more number of users and under different SNR scenarios. We limit ourselves to symmetric scenario (this is for convenience, the general case can be handled similarly). In sensor networks sensors in a neighbourhood which transmit to a cluster-head will approximately satisfy this condition.

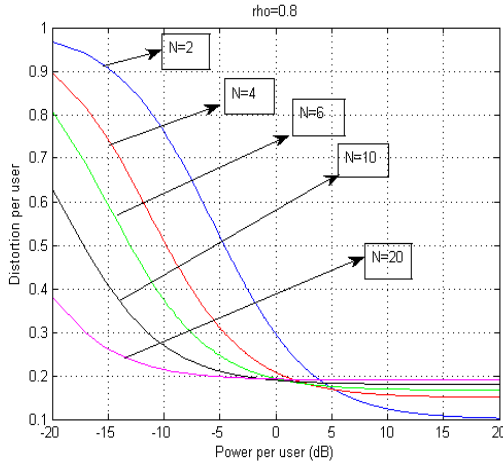


Fig. 2. Distortion for AF,  $\rho = 0.8$ .

Consider jointly Gaussian  $(U_1, U_2, \dots, U_N)$  having mean zero vector with  $E(U_i, U_j) = \rho$ ,  $i \neq j$  and  $E[U_i^2] = 1$ . Let all the nodes be (average) power constrained to  $P$ . Let  $D_{AF}(N, P)$  denote the distortion per user. For this scheme

$$D_{AF}(N, P) = 1 - \frac{P(1 + (N-1)\rho)^2}{NP(1 + (N-1)\rho) + \sigma^2}. \quad (1)$$

Thus

$$\lim_{N \rightarrow \infty} D_{AF}(N, P) = 1 - \rho, \quad (2)$$

$$\lim_{P \rightarrow \infty} D_{AF}(N, P) = \frac{N-1}{N}(1 - \rho). \quad (3)$$

From (2) we see that distortion per user tends to  $1 - \rho$  for all  $P$ . Thus correlation reduces the asymptotic mean distortion. Also from (3) we see that, at high SNR, distortion per user increases from  $(1 - \rho)/2$  for the two user case to  $1 - \rho$  as  $N \rightarrow \infty$ .

We plot  $D_{AF}(N, P)$  for  $\rho = 0.8$  in Fig. 2. We find that for low SNR as  $N$  increases  $D_{AF}(N, P)$  decreases. But at high SNR as  $N$  increases  $D_{AF}(N, P)$  increases. This happens because of the trade-off between the interference caused by many users and the beamforming gain (because of the correlation in the observations). This trade-off depends on the SNR. The cut-off where this change (decrease to increase) occurs depends on  $\rho$ . The cut-off increases with  $\rho$ . For  $\rho = 0.8$  and for  $N \leq 10$  the cutoff could be taken as  $3\text{dB}$ . Thus, for  $n \leq 10$ , for  $P \leq 3\text{dB}$  distortion  $D(N, P)$  decreases with  $N$ .

We have seen similar behaviour for the system with side information at the decoder. For more information on how we use side information, see Section V.

## B. SB-TDMA

In this coding scheme the source output  $U_i^n \triangleq (U_{i1}, \dots, U_{in})$ ,  $i = 1, \dots, N$  is vector quantized to  $W_i^n$  by sensor node  $S_i$ . The  $W_i^n$ 's are Slepian-Wolf encoded. The encoded data is sent over the channel using a TDMA scheme.  $W_i^n$ 's are losslessly obtained at the decoder and then  $U_i^n$ 's are estimated. This scheme corresponds to the scheme used in [9], [7].

We compare the performance of SB-TDMA with AF through computations. Fig. 3 gives the comparison for  $N = 2$  and  $N = 3$ . We see that distortions for AF are lower at low SNRs than for SB-TDMA.

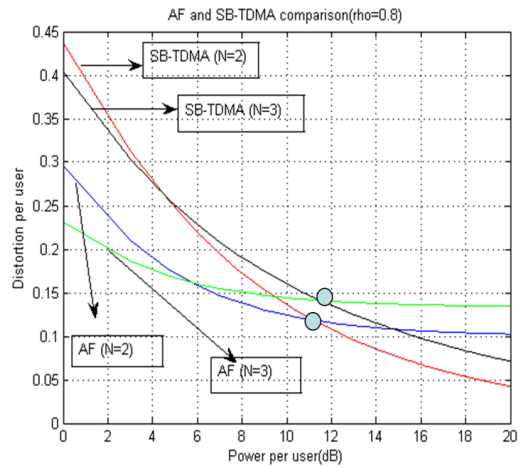


Fig. 3. Comparison of AF and SB-TDMA,  $\rho = 0.8$

Let  $D_{SB-TDMA}(N, P)$  be the distortion per user for the SB-TDMA scheme. Also let  $SNR_{Th}$  be the SNR upto which AF is better than SB-TDMA. From Fig. 3 we find that for  $N = 2$ ,  $SNR_{Th} = 11\text{dB}$  and it increases to  $11.8\text{dB}$  for  $N = 3$ . We have done computations for several other examples also (not provided due to lack of space) and observe that this happens in general: as the number of users increases, AF performs better than SB-TDMA for a larger SNR region, i.e.,  $SNR_{Th}$  increases with  $N$ . This is due to the larger beamforming gain achieved by AF for more number of users.

## IV. TRANSMISSIONS FROM A RELAY NODE

Figure. 4 represents a typical scenario when the relay nodes (cluster heads) transmit over a GMAC.  $U_1, \dots, U_{N/2}$  data streams are transmitted by the first relay node and  $U_{N/2+1}, \dots, U_N$  by the second. The transmitted symbol (in one channel use) by relay  $i$ ,  $i = 1, 2$  is  $X_i$ . We study the performance of both AF and SB-TDMA over such a network for the symmetric case. In AF  $U_1, \dots, U_{N/2}$  are linearly combined such that  $E[X_1^2] = NP/2$ . Similarly  $U_{N/2+1}, \dots, U_N$  are linearly combined such that  $E[X_2^2] = NP/2$ . This ensures

that the total power is same as in Section III (this is to compare with the results in section III). In SB-TDMA scheme the sources  $U_1, \dots, U_N$  are vector quantized and then Slepian-Wolf encoded [24]. For transmission of these encoded sources, half of the time node 1 transmits and rest of the time node 2 transmits thereby converting the MAC to orthogonal channels. When node 1 transmits, the Slepian-Wolf encoded outputs are again transmitted in a TDMA fashion. Similarly for node 2. The estimates  $(\hat{U}_1, \dots, \hat{U}_N)$  are obtained from  $Y$  and are given by  $E[U_1, U_2, \dots, U_N|Y]$ .

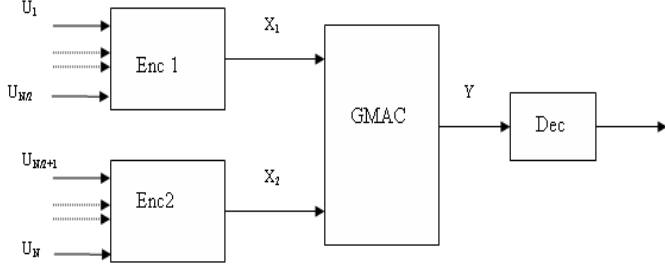


Fig. 4. Typical Relay node

For AF the distortion per user is given by,

$$D_{RAF}(N, P) = 1 - \frac{2P(1 + (N-1)\rho)^2}{2(NP + \sigma^2)(1 + (N/2 - 1)\rho) + N^2P\rho} \quad (4)$$

From (1) and (4) we see that  $D_{AF}(N, P) < D_{RAF}(N, P)$  if  $N > 2$  and  $D_{AF}(2, P) = D_{RAF}(2, P)$ . Also from (4),

$$\lim_{N \rightarrow \infty} D_{RAF}(N, P) = 1 - \rho,$$

$$\lim_{P \rightarrow \infty} D_{RAF}(N, P) = 1 - \frac{2(1 + (N-1)\rho)^2}{2N(1 + (\frac{N}{2} - 1)\rho) + N^2\rho},$$

$$\lim_{N \rightarrow \infty} \lim_{P \rightarrow \infty} D_{RAF}(N, P) = 1 - \rho.$$

Fig.5 plots  $D_{RAF}$  for various  $N$  and  $P$  values. The effect of side information is similar as discussed in Section III.

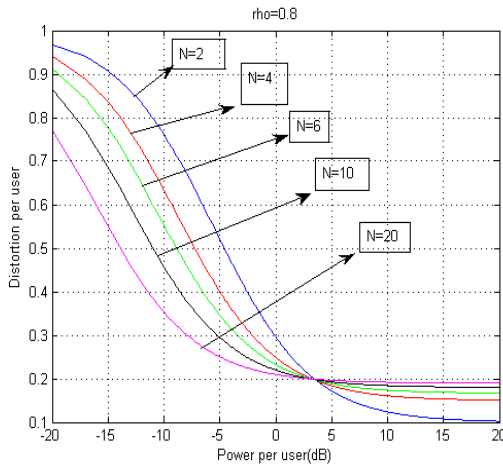


Fig. 5. Distortion per user for AF, relay nodes,  $\rho = 0.8$ .

Comparing Figs. 2 and 5 we find that for the same power in the system the relay node performs worse than the data node at low SNR. At high SNR the performance of both nodes are close. These are in accordance with the theoretical results above. The difference between the two increases with  $\rho$ . The difference between  $D_{AF}(N, P)$  and  $D_{RAF}(N, P)$  is due to the power constraint. The power available for each component at the input of relay node is less than that of the data node even when  $E[X_i^2]$  is same. Furthermore, the diversity of the data node is larger than the relay node. A similar conclusion holds for the case with decoder side information.

Fig.6 compares the performance of AF and SB-TDMA for  $N = 4, 6$  over the relay node. It is found from the figure that AF performs better than SB-TDMA over a large range of SNRs (say till 14 dB for  $N=4$ ). This cut-off SNR increases with increase in  $N$  and also with increase in  $\rho$ .

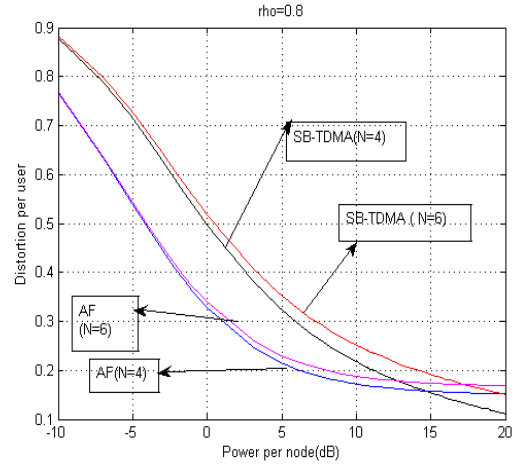


Fig. 6. Comparison of AF and SB-TDMA,  $\rho = 0.8$

## V. AF AND SB-TDMA OVER HIERARCHICAL NETWORKS

In the previous sections we have shown that AF performs better than SB-TDMA for the network components (single hop). Now we study the same comparison for the hierarchical networks such as in Fig 1.

In AF, in cluster  $C_i$ , each sensor  $S_{ij}$ ,  $j = 1, \dots, M_i$  amplifies its observation  $U_{ij}$  to the power constraint  $P_{ij}$  and transmits it over the GMAC to the cluster head  $H_i$ . The received signal  $Y_i$  at the cluster head is linearly combined with the cluster head's observation  $U_i$  and transmitted over the GMAC to the sink. The coefficient for the linear combination for observations  $U_i$  is  $a_i$  and for received signals  $Y_i$  is  $b_i$ .  $a_i$  and  $b_i$  are chosen through an optimization scheme which minimizes the sum of distortions at the sink and also satisfies the power constraint  $P_i$  at each cluster head.

For simplicity of the analysis we assume a symmetric network where the variances of all the sources are same ( $= \sigma^2$ ) and also the power constraints  $P_{ij} = P$  for all  $i$  and  $j = 1, \dots, M_i$ . We assume a homogenous network where the cluster head also has the same power (i.e.  $P_i = P$  for all  $i$ ).

Symmetry is taken for convenience and is not a restriction of the approach.

### A. Two nodes per cluster head

Nodes  $S_{11}$  and  $S_{12}$  transmit  $X_{11}$  and  $X_{12}$  to  $H_1$ , where  $X_{1j} = \sqrt{P}U_{1j}$ ,  $j = 1, 2$ . The received signal at  $H_1$  is  $Y_1 = X_{11} + X_{12} + V_1$ . Node  $H_1$  linearly combines  $Y_1$  and  $U_1$  to form  $X_1$ , i.e.,  $X_1 = b_1Y_1 + a_1U_1$ . Similarly we operate at nodes  $S_{21}, S_{22}$  and  $H_2$  and obtain  $X_2 = b_2Y_2 + a_2U_2$ . These  $X_i$ ,  $i = 1, 2$  are transmitted to the sink. The received signal at the sink is  $Y = X_1 + X_2 + V$ . We take  $E[V] = E[V_i] = 0$  and  $var(V_i) = var(V) = 1$  for all  $i$ . Given  $Y$  and the sink node observation  $U$  the sources are estimated as  $E[U_{11}, U_{12}, U_{21}, U_{22}, U_1, U_2|Y, U]$ .

The best  $\{a_i, b_i\}$  are found by formulating an optimization procedure such that the sum of distortions is minimized at the sink node subject to the power constraints.

In SB-TDMA  $U_{1j}^n$ , is vector quantized to  $W_{1j}^n$ . The  $W_{1j}^n$ s are Slepian-Wolf encoded and sent over orthogonal channels to  $H_1$ . The rates at which the quantizers operate are found from the region ([22]),

$$\begin{aligned} I(U_{11}; W_{11}|W_{12}, U_1) &< \frac{1}{4} \log\left(1 + \frac{2P}{N}\right), \\ I(U_{12}; W_{12}|W_{11}, U_1) &< \frac{1}{4} \log\left(1 + \frac{2P}{N}\right), \\ I(U_{11}, U_{12}; W_{11}, W_{12}|U_1) &< \frac{1}{2} \log\left(1 + \frac{2P}{N}\right). \end{aligned} \quad (5)$$

In the above scheme global network knowledge is not required. Only local knowledge (statistics of  $U_i$  that are transmitted on the same MAC) is assumed. At  $H_1$ ,  $U_{11}^n, U_{12}^n$  are estimated as  $\hat{U}_{1i}^n$ ,  $i = 1, 2$  using  $W_{11}^n, W_{12}^n$  and  $U_1^n$  as  $E[U_{11}^n, U_{12}^n|W_{11}^n, W_{12}^n, U_1^n]$ .

Similarly we obtain  $\hat{U}_{21}^n, \hat{U}_{22}^n$  at  $H_2$ . For symmetric case let  $d_i$  be the distortion incurred in  $U_{ij}$ . Then we can write  $\hat{U}_{ij} = U_{ij} + d_i$ ,  $i = 1, 2; j = 1, 2$ .

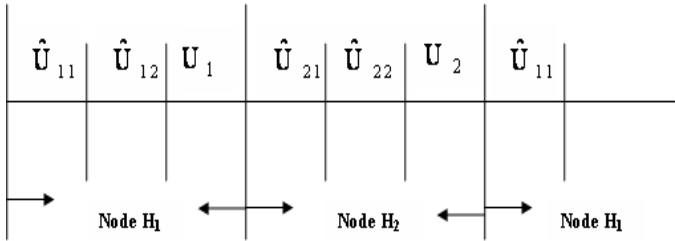


Fig. 7. Transmissions from  $H_1$  and  $H_2$

Node  $H_1$  vector quantizes  $(\hat{U}_{11}^n, \hat{U}_{12}^n, U_1^n)$  to  $(Z_{11}^n, Z_{12}^n, Z_{13}^n)$ . These are then Slepian-Wolf encoded and then transmitted through a TDMA scheme as shown in Fig.7. Node  $H_2$  operates in the same way. The rates at which the quantizers operate are given from a region similar to (5) with the 6 inequalities given by

$$I(U_A; Z_A|Z_{A^c}, U) < \sum_{i=1}^{|A|} \frac{1}{2} \log\left[\left(1 + \frac{2P}{N}\right)^{\frac{|A|}{6}}\right], \quad (6)$$

where  $A \subset S = \{11, 12, 21, 22, 13, 23\}$  and  $|A|$  denotes the number of elements in  $A$ .

These conditions ensure that  $Z_{ij}^n$  are losslessly recovered. We estimate the sources as  $E[U_{ij}^n, U_i^n|Z_{11}^n, \dots, Z_{22}^n, U^n]$ ,  $i = 1, 2; j = 1, 2$ . The aim is to minimize the sum of distortions at the sink.

Now we compare the two schemes for different  $P$ . The sources considered have zero means and unit variances. Let  $C_1 = \{U_{11}, U_{12}, U_1\}$  and  $C_2 = \{U_{21}, U_{22}, U_2\}$ . Let  $\rho_1$  denote the correlation between any two elements in  $C_1$ . Same holds for  $C_2$  also. Let  $\rho_2$  denote the correlation between an element in  $C_1$  and an element in  $C_2$ . The correlation between  $U$  and any element in  $C_1$  or  $C_2$  is  $\rho_3$ . Also,  $\rho_4 = E[UU_1] = E[UU_2]$ .

Consider the case when  $\{\rho_i, i = 1, 2, 3, 4\} = \{0.8, 0.6, 0.1, 0.3\}$ . This is under the assumption that correlation decreases with distance. Comparison of the two schemes is given in Table I.

TABLE I  
MINIMUM SUM OF DISTORTIONS

$P(\text{dB})$	$D_{AF}$	$D_{SB-TDMA}$
0	2.70	3.5
3	2.22	2.95
7	1.84	2.3
10	1.7	1.93
13	1.62	1.63
14.7	1.59	1.48

From Table I we see that AF outperforms SB-TDMA till about 13 dB of SNR. If correlations are reduced then the cut off SNR up to which AF performs better than SB-TDMA is reduced.

When the nodes generate independent data, AF performs better than SB-TDMA only below -6dB of SNR.

From computations we have found that similar conclusions as in Table. I hold when the cluster head has more power.

### B. Three nodes per cluster head

Now we consider the case with 3 clusters and 3 sensor nodes in each cluster. Each  $U_{ij}, U_i$  has zero mean, unit variance with the following correlation structure. The correlation between any two sources in  $C_1, C_2$  or  $C_3$  is  $\rho_1$ . The correlation between a source in  $C_1$  and another in  $C_2$  is  $\rho_2$ . Similarly for  $C_2$  and  $C_3$ . The correlation between an element in  $C_1$  and an element in  $C_3$  is  $\rho_3$ . The correlation between  $(U_1, U_3)$ ,  $(U_2, U_3)$  or  $(U_3, U_1)$  is  $\rho_4$ . Also the correlation between  $U$  and any element of  $C_1, C_2$  or  $C_3$  is  $\rho_5$ .

Let  $\{\rho_i, i = 1, \dots, 5\} = \{0.8, 0.6, 0, 0.4, 0\}$ . The comparison of AF and SB-TDMA with respect to the minimum sum of distortions achieved is given in Table II.

From Table II we find that AF performs well for such networks at low SNR.

### C. Five nodes per cluster head

Let  $C_1 = \{U_{11}, U_{12}, U_{13}, U_{14}, U_{15}, U_1\}$ ,  $C_2 = \{U_{21}, U_{22}, U_{23}, U_{24}, U_{25}, U_2\}$ . The correlation structure is as defined in subsection V-A. For,  $\{\rho_i, i = 1, 2, 3, 4\} =$

TABLE II  
MINIMUM SUM OF DISTORTIONS

$P(\text{dB})$	$D_{AF}$	$D_{SB-TDMA}$
0	5.22	7.08
3	4.58	5.9
5	4.32	5.27
7	4.16	4.66
9	4.05	4.12
10	4.01	3.87

{0.8, 0.6, 0.5, 0.2}. We compare AF and SB-TDMA in Table III.

TABLE III  
MINIMUM SUM OF DISTORTIONS

$P(\text{dB})$	$D_{AF}$	$D_{SB-TDMA}$
0	5.27	6.64
3	4.39	5.53
5	4.00	4.88
7	3.73	4.30
10	3.49	3.66
12	3.41	3.29

From Table. III we see that AF performs even with 5 nodes in a cluster upto 10 dB. This is also inline with the conclusions of the single hop case as discussed in Section III.

When the network depth is increased the performance of AF degrades a little due to the amplification of noise at each hop. However even for a 3 hop- network AF performs well for a reasonable range of SNR (results not provided due to lack of space).

## VI. CONCLUSIONS

This paper studies the performance of Amplify and Forward transmission for hierarchical networks. The effectiveness of AF for the network building blocks (i.e. data nodes and relay nodes) are studied. The scaling laws for AF with number of users and power is also studied. It is also shown that AF performs well over a two level hierarchical network as compared to the SB-TDMA.

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