Analysis of DualCUSUM: a Distributed Energy Efficient Algorithm for Change Detection

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***Abstract*—We analyse a simple and energy efficient distributed Change Detection scheme for sensor networks based on Page's CUSUM algorithm. Each sensor runs CUSUM and transmits only when the CUSUM is above some threshold. The transmissions from the sensors are fused at the physical layer. The channel is modeled as a Multiple Access Channel (MAC) corrupted with IID noise. The fusion center which is the global decision maker, performs another CUSUM to detect the change. This scheme was first proposed in [2]. In this paper, we generalize the algorithm to also include nonparametric CUSUM and provide a unified analysis.**

I. Introduction

A sensor network is often deployed for detecting a change in the distribution of a random field, e.g., intrusion detection in a geographical area. We propose energy-efficient distributed algorithms for such a network which minimize the detection delay subject to realistic constraints on false alarm. In such a set up, the sensor network is divided in clusters and the sensors in each cluster sense observations and transmit the processed information to the cluster head (called the fusion center in the following).

It is assumed that the entry of an intruder will statistically affect the observations taken by all the sensors in a cluster, simultaneously. Thus, the algorithms developed for the detection of change in the distribution of a sequence of observations ([12], [18], [21]) can be useful.

The centralized version of the change detection problem where sensors send raw observations to the fusion center is well studied. Various efficient algorithms have been proposed ([12], [18]), and their optimality properties have been shown ([7], [9], [11], [13], [18]).

In sensor networks, change detection will be done via observations at many sensors. Also, each node has limited computational resources and little power. Thus, we need to consider *decentralized* change detection problems which are computationally simple and require minimal transmission.

The problem of decentralized detection is nicely surveyed in [3], [20] and [22]. According to [20] various information structures are possible in the decentralized setting. In the Bayesian formulation ([18]), where the distribution of the time of change is known, the problem is intractable except for the quasi-classical *case E* in [20]. In the non-Bayesian formulation of the problem Mei ([9]) has shown asymptotic Min-Max optimality of CUSUM based schemes (originally developed in [12]). A comparison of various distributed detection schemes under Bayesian and non-Bayesian settings is reported in [9], [10], and [19] .

In all the above decentralized strategies the change detection problem is solved at the link layer, by assuming a perfect physical layer (reliable communication) and energy is conserved by sending fewer bits. Thus a few issues omitted in these studies are: The energy used and the delay incurred in transmission of information from the sensors to the fusion node.

In [14] and [23], the second effect is addressed. In [14], the network delay is specifically modeled and optimized. In [23] all the sensors transmit at the same time and physical layer fusion is exploited.

In [2], we have proposed an energy efficient scheme called DualCUSUM. Our algorithm has the following desirable features. The sensors use CUSUM locally and transmit information to the fusion center only when they detect a change. Since change usually occurs very rarely, this saves a lot of energy in transmission. Also, we use physical layer fusion (as in [23]): All nodes transmit at the same time. This minimizes the time needed to transmit at the MAC. But unlike in [23], we do not require feedback from the fusion node to the sensor nodes. This reduces delay and energy at the fusion node and uses less computations. Furthermore, using CUSUM at the fusion node we exploit all the past information.

Physical layer fusion requires phase, frequency and time synchronization of different nodes. This is feasible in sensor networks. However, if one does not provide for such synchronization, our algorithm can be used without physical layer fusion (using other MAC layer protocols, e.g., TDMA). Due to other features mentioned above, it will still provide good performance.

Although this algorithm has many desirable features, there is one practical limitation: to use CUSUM one needs the distribution of observations before and after change at each sensor node. This may not be a realistic assumption in many cases. For example, there can be random time varying fading in the wireless channels in sensor networks. Another example is a Constant False Alarm Rate (CFAR) distributed detection system ([3]) where the clutter power is usually unknown. Thus, in this paper we generalize our algorithm to also include nonparametric CUSUM, if needed, in our overall algorithm. Our analysis also throws light on why CUSUM performs better than nonparametric CUSUM.

The paper is organized as follows. We explain the model and introduce the algorithm in Section II. Section III analyzes the performance of the algorithm and provides comparison with simulations. Section IV concludes the paper.

II. Model and Algorithm

Let there be L sensors. Let $X_{k,l}$ be the observation made at sensor l at time k. Sensor l transmits $Y_{k,l}$ at time k after processing $X_{k,l}$ and its past observations. The distribution of the observations at each sensor changes simultaneously at a random time T with a known distribution. Before the change $\{X_{k,l}, k \geq 1\}$ are independent and identically distributed (iid) with density f_0 and after the change with density f_1. The expectation under f_i will be denoted by E_i, $i = 0, 1$, and P_∞ and P_1 denote the probability measure under no change and when change happens at 1, respectively.

These assumptions have been made in almost all decentralized detection studies ([19], [20], [22]).

The objective of the fusion center is to detect this change as soon as possible at time τ (say) using the messages transmitted from all the L sensors, subject to an upper bound on the probability of False Alarm $P_{FA} = P(\tau < T)$ and the average energy used. Then, the general problem is:

$$\min E_{DD} \stackrel{\triangle}{=} E[(\tau - T)^+], \text{ Subj to } P_{FA} \leq \alpha \quad \text{and}$$
$$E\left[\sum_{k=1}^{\tau} Y_{k,l}^2\right] \leq \mathscr{E}_0, 1 \leq l \leq L. \tag{1}$$

Our algorithm in [2] does not provide an optimal solution to (1) but uses several desirable features to provide better performance than the algorithms we are aware of. We reproduce the algorithm below for an easy reference.

Sensor l uses CUSUM,

$$\begin{aligned} W_{k,l} &= \max\left(0, W_{k-1,l} + \log\left[f_1\left(X_{k,l}\right)/f_0\left(X_{k,l}\right)\right]\right), \quad (2)\\ W_{0,l} &= 0, \quad 1 \leq l \leq L. \end{aligned}$$

Sensor l transmits $Y_{k,l} = b 1_{\{W_{k,l} > \gamma\}}$. Here 1_A denotes the indicator function of set A and b is a design parameter. Physical layer fusion (as in [23]) is used to reduce transmission delay, i.e., $Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_{MAC,k}$. Finally, Fusion center runs CUSUM:

$$F_k = \max\left\{0, F_{k-1} + \log\frac{g_I(Y_k)}{g_0(Y_k)}\right\}; \quad F_0 = 0, \tag{3}$$

where g_0 is the density of $Z_{MAC,k}$ and g_I is the density of $Z_{MAC,k} + bI$, I being a design parameter. The fusion center declares a change at time $\tau(\beta, \gamma, b, I)$ when F_k crosses a threshold β: $\tau(\beta, \gamma, b, I) = \inf\{k : F_k > \beta\}$.

In the absence of $Z_{MAC,k}$, it has been shown ([9]) that at the fusion center, it is asymptotically optimal to declare a change as soon as all the sensors transmit, i.e., $\sum_{l=1}^{L} Y_{k,l} = Lb$. But in the presence of MAC noise, it is not possible for the fusion center to know how many sensors are transmitting at any time.

Multiple values of (β, γ, b, I) will satisfy both the false alarm and the energy constraint in (1) and one can minimize E_{DD} over this parameter set. In Section III we obtain the performance of DualCUSUM for any parameter values (β, γ, b, I) which then can be used in the optimization algorithm developed in [2] to solve efficiently the optimization problem:

$$(\beta^*, \gamma^*, b^*, I^*) = \arg\min_{(\beta,\gamma,b,I)} E_{DD}(\beta, \gamma, b, I) \tag{4}$$

subj to $P(\tau(\beta, \gamma, b, I) < T) \leq \alpha$, energy $\mathscr{E}_{avg}(\beta, \gamma, b, I) \leq \mathscr{E}_0$.

This algorithm was compared with that of [23] via simulations in [2] and was shown to perform much better. The performance tends to improve as P_{FA} decreases. It is shown in [23] that their algorithm performs better than some other algorithms in literature.This motivates us to study this algorithm.

DualCUSUM, as the original CUSUM itself, has a strong limitation. It requires the exact knowledge of f_0 and f_1. This information will be available apriori to varying degrees in a practical scenario depending upon the type of uncertainty in f_0, f_1, different algorithms/variations on CUSUM are available ([5], [6]). One common algorithm, called nonparametric CUSUM is to replace (2) by

$$W_{k+1,l} = \max\{0, W_{k,l} + X_{k,l} - D\}, \tag{5}$$

where, D is an appropriate constant such that $E[X_{k,l} - D]$ is negative before change and positive after change. For Gaussian and exponential distributions, nonparametric CUSUM becomes CUSUM for some appropriate D and scaling.

If at the fusion node g_0 is not known (in our algorithm $g_I(x) = bI + g_0(x)$), then one can use (5) even at the fusion node.

In the following we will provide analysis of a generalized class of algorithms where at the sensor nodes and at the fusion node we use the algorithm,

$$W_{k+1} = \max\{0, W_k + Z_{k+1}\}, \tag{6}$$

where, $\{Z_k\}$ is an iid sequence. If we use CUSUM at a sensor node then $Z_k = \log f_1(X_k)/f_0(X_k)$; for non parametric CUSUM, $Z_k = X_k - D$. It is known that CUSUM performs better than nonparametric CUSUM ([5], [6]). This is of course the penalty one pays for having less information. Our analysis will provide additional insight on why CUSUM performs better than nonparametric CUSUM.

III. Analysis

We analyze the performance of our (generalized) DualCUSUM algorithm in this section. First, we compute the false alarm probability P_{FA} and then E_{DD}. The idea is to model the times at which the CUSUM $\{W_k\}$ at the local sensors, crosses the threshold γ (till Section III-B we will be dealing with these processes at local nodes. Thus the processes $\{W_{k,l}\}$ and $\{Z_{k,l}\}$ will be denoted without the subscript l). At these times the local nodes will transmit to the fusion node. For low values of P_{FA} these excursions above γ should be rare. Also these excursions happen in batches because when W_k exceeds γ, it is very likely that it will exceed γ in the next step also (in Figure 1 η is a batch). We obtain approximations for the probability of false alarm during these batches.

Often it is said that light tailed distributions may provide better system behaviour than the heavy tailed (see e.g., [4] for a discussion in case of networks). In the present context this issue will come up several times. First we show via examples that CUSUM has the interesting property that it transforms a large class of heavy tailed distributions into light tailed distributions.

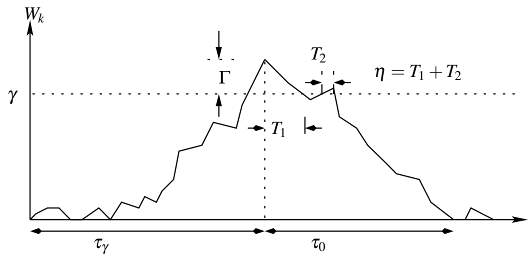

Fig. 1. Excursions of W_k above γ can be approximated by a compound Poisson process.

Let $f_1(x) \sim N(\mu, \sigma^2), f_0(x) \sim N(0, \sigma^2)$ where $N(a,b)$ denotes Gaussian distribution with mean a and variance b. Then, $\log(f_1(X)/f_0(X)) = \mu X/\sigma^2 - \mu^2/2\sigma^2$.
Hence $Z = log(f_1(X)/f_0(X))$ continues to have light tail.

Next consider the case when f_0 and f_1 have Pareto distribution $Kx_m^K/x^{(K+1)}$ for $x \geq x_m$, where $x_m > 0$ and for f_0, $K = (\alpha+1)$ and for f_1, $K = \alpha$ where α is a positive constant. Then, $P_\infty[Z_k > z] \sim e^{-z\alpha}$. Thus even when f_0, f_1 have heavy tails, Z has light.

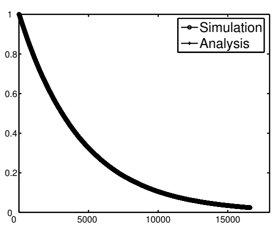

Fig. 2. Complementary CDF of τ_γ for $Z_k \sim N(-0.3, 1)$ via Integral equation (10) and $\gamma = 10$.

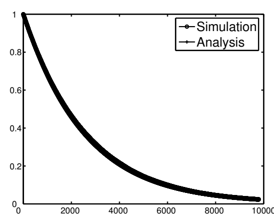

Fig. 3. Complementary CDF of τ_γ via (10) for Pareto $K = 2.1$, $EZ_k = -0.3$, $var(Z_k) = 1$, $\gamma = 10$.

This very important property of log likelihood seems to have escaped the attention of investigators before. This makes CUSUM perform better than the nonparametric CUSUM. Also, because of this, the analysis provided in [2] works quite well for a large class of distributions with CUSUM but will not for nonparametric CUSUM. The rest of the paper provides the analysis which works for both.

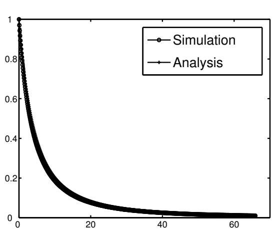

Fig. 4. Complementary CDF of Γ for Pareto $K = 2.1$, $EZ_k = -0.3$ and $var(Z_k) = 1$ and $\gamma = 8$

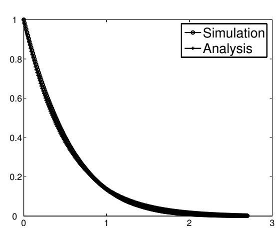

Fig. 5. Complementary CDF of Γ for $Z_k \sim n(-0.3, 1)$ and $\gamma \geq 6$

A. Behavior of W_k under P_∞

The process $\{W_k\}$ is a reflected random walk with negative drift under P_∞. Figure (1) shows a typical sample path for $\{W_k\}$. The process visits 0 (regenerates) a finite number of times before it crosses the threshold γ at,

$$\tau_\gamma \overset{\triangle}{=} \inf\{k \geq 1 : W_k \geq \gamma\}. \tag{7}$$

We call τ_γ the *First Passage Time (FPT)*. The *overshoot* $\Gamma = W_{\tau_\gamma} - \gamma$. Let

$$\tau_0 \overset{\triangle}{=} \inf\{k : k > \tau_\gamma; W_k \leq 0\} - \tau_\gamma \quad \text{and}$$
$$\eta = \#\{k : W_k \geq \gamma; \tau_\gamma \leq k \leq \tau_\gamma + \tau_0\}. \tag{8}$$

It has been shown in [15] that the point process of exceedances of γ by W_k, converges to a compound Poisson process as $\gamma \to \infty$. The points appear as clusters. The intervals between the clusters have the same distribution as that of τ_γ in (7) and the distribution of η in (8) gives the distribution of the size of the cluster, i.e., the batch of the compound Poisson process. Since, one has to choose large values of γ to keep P_{FA} small, a batch Poisson process provides a good approximation in our scenario.

In the following we give results for the distribution of τ_γ, overshoot Γ, and the distribution of the batch η which will be used in computing P_{FA}. From the compound Poisson process approximation mentioned above,

$$\lim_{\gamma \to \infty} \mathbf{P}_\infty\{\tau_\gamma > x\} = \exp(-\lambda_\gamma x), \ x > 0, \tag{9}$$

where, λ_γ a positive constant.

Various methods are suggested in literature to compute λ_γ. In [2] a formula was used which is computable for Gaussian distributions only.

Solving integral equations obtained via renewal arguments is a well known method of obtaining the mean of FPT ([16]). Let $L(s)$ be the mean FPT with $W_0 = s$ and hence $\lambda_\gamma = 1/L(0)$. Then from renewal arguments:

$$L(s) = 1 + L(0)F_Z(-s) + \int_0^\gamma L(y) f_Z(y - s) dy. \tag{10}$$

This method gives exact value of λ_γ but can be computationally intensive for a large γ. An algorithm provided in [8] can be used to compute (10) efficiently. Figures (2) and (3) compare the exponential distribution with λ_γ obtained from (10) with simulations when Z_k is Gaussian and Pareto with $K = 2.1$ and $EZ_k = -0.3$, $Var(Z_k) = 1$ and $\gamma = 10$. One sees a very good match with simulations.

One sees from Figures (2) and (3) that the heavy tailed distributions have smaller (in probability) τ_γ than light tailed distribution (Gaussian). The difference increases with γ. To emphasize this point, in Table (I) we provide $E[\tau_\gamma]$ for Pareto with $K = 2.1$ and Gaussian distribution. Since CUSUM converts a heavy tailed X_k into a light tailed Z_k centralized CUSUM performs better than centralized nonparametric CUSUM.

Let $F_\Gamma(x, u)$ be the distribution of Γ when $W_0 = u$. Then, the following integral equation, similar to (11), can be written:

$$\begin{aligned} F_\Gamma(x,u) &= F_Z(-u)F_\Gamma(x,0) + \int_0^\beta f_Z(y) F_\Gamma(x, u+y) dy \\ &+ \bar{F}_Z(\beta - u) P(Z \leq \beta - u + x | Z > \beta - u). \end{aligned} \tag{11}$$

This is computationally expensive. We have developed some asymptotic approximations to obtain faster algorithms, which will be presented in a future work.

Now we obtain the distribution of the batch.

1) Distribution of batch for heavy tail: From Theorem 2.4 of [1], the batch size distribution for subexponential Z (belonging to the Maximum Domain of Attraction of a Frechet distribution H with parameter α) satisfies

$$\frac{E[Z]}{\omega(\gamma)}\eta \xrightarrow{d} Y_\alpha, \tag{12}$$

as $\gamma \to \infty$, where $\omega(\gamma) = E[Z-\gamma|Z>\gamma]$ and Y_α has distribution P_α.

2) Distribution of batch for light tail: Let $G_j(x)$ be the conditional batch distribution, $G_j(x) = P(\eta \le j|W_{\tau_\gamma} = \gamma + x)$, when the overshoot is x. $G_j(x)$ was obtained in [2] by the Brownian Motion (BM) approximation of $\{W_k\}$. In [2], x was replaced with $E[\Gamma]$. This provides a good approximation only for Gaussian distribution. Thus in general for a light tailed distribution, we now use the exponential distribution:

$$P(\eta \le j) = \int_0^\infty G_j(x)\frac{1}{R(0)}exp(-\frac{x}{R(0)})dx, \tag{13}$$

where, $R(0)$ is the mean overshoot which can be computed from integral equation like (11).

We use equation (13) to plot the distribution of η for Laplace Z_k and compare it with the corresponding simulations in Figure (6). The plot shows that equation (13) indeed gives a very good approximation to the batch size.

B. False Alarm Analysis

The false alarm in DualCUSUM can happen in two ways: one within a batch (we denote its probability by $\tilde{p}$) and another outside it, i.e., due to $\{Z_{MAC,k}\}$. We study them in the following. From the compound Poisson approximation and independence of CUSUM processes at each sensor, under symmetric statistical assumptions, the inter-arrival time of the batches in the system (at the fusion center) is exponentially distributed with rate $L\lambda_\gamma$. Since we are interested in the region of low values of false alarm probability and change, the following approximations seem quite realistic:

1) The batch size η is much smaller as compared to the time of change, i.e. $P(\eta << T) \approx 1$.
2) Batches of different sensors do not overlap in time.

However, we have been able to verify these assumptions via simulations only for light tailed Z_k. Thus, we first provide the analysis for the light tailed case and then consider subexponential distributions. Of course the P_{FA} outside a batch (due to $Z_{MAC,k}$) is not affected by the distribution of Z_k.

Let, ψ be the number of batches before change. With $\{W_k\}$ and T being independent, the number of batches appearing before the time of change say is a Poisson random variable with parameter $L\lambda_\gamma i$, when $T = i$. The overall false alarm probability P_{FA} at the fusion center can be written as,

$$P_{FA} = \sum_{i=1,j=0}^{\infty} P(FA|T=i;\psi=j)P(\psi=j|T=i)P(T=i). \tag{14}$$

In Section given below, we will show that the time to FA outside a batch is exponentially distributed with parameter λ_0. Therefore,

$$\begin{aligned} & P(FA|T=i;\psi=j) \\ \approx\ & 1 - P(\{\text{ No FA in } j \text{ batches}\} \\ & \cap\{\text{ No FA in } i \text{ observations}\}|T=i;\psi=j) \\ \approx\ & 1-(1-\tilde{p})^j e^{-\lambda_0 i}. \end{aligned} \tag{15}$$

If $T \sim \text{Geom}(\rho)$, from (14) and (15) we get:

$$P_{FA} = 1 - \frac{e^{-(\lambda_0+\lambda_\gamma L\tilde{p})}\rho}{1-e^{-(\lambda_0+\lambda_\gamma L\tilde{p})}(1-\rho)}. \tag{16}$$

False Alarm within a Batch:

Next we compute $\tilde{p}$. First we consider light tailed Z_k. Then the approximation mentioned in the beginning of this section holds. The false alarm probability within a batch, $\tilde{p}$ can be computed as, $\tilde{p} \approx \sum_{i=1}^{\infty} P(\eta=i)P(\text{FA }|\eta=i)$, where η has the distribution (13) for light tailed Z_k. $P(\text{FA }|\eta=i)$ represents the probability of FA (CUSUM at the fusion center crossing β) in i transmissions when, $Y_k = b + \sum_{l=1}^{L} Y_{k,l} + Z_{MAC,k}$. If τ_β is the FPT variable at the fusion center, then, $P(\text{FA }|\eta=i) = P(\tau_\beta \le i)$. The mean EZ_k can be positive or negative based on the choice of D and I. Since η is small for negative drift, we use integral equations for distribution of FPT. Efficient algorithms are available to compute them ([8]). If $p(n|x)$ is the probability that $\tau_\beta = n$ given $W_0 = x$, then,

$$p(n|x) = \int_0^\beta p(n-1|y)f_Z(y-x)dy + p(n-1|0)F_Z(-x). \tag{17}$$

We use this to compute $P(\text{FA }|\eta=i)$ for light tailed Z_k. Table II gives the comparison of the P_{FA} values obtained via (16) and simulations for light tailed distributions Gaussian and Laplace.

For subexponential tails the above technique provides poor approximations. Recently we developed a better approximation for $\tilde{p}$ for this case. Due to lack of space, we donot provide details.

The results are summarized in Table III. One can see that the approximation is indeed good.

False Alarm outside a Batch: In the absence of any transmission from the sensors, $Y_k \sim N(0,\sigma_{MAC}^2)$. Hence, F_k has negative drift. Thus the time to first reach β, i.e., time till FA, is approximately exponentially distributed with parameter λ_0 which can be obtained from (10).

C. Computation of E_{DD}

The detection delay, E_{DD}, can be written as,

$$\begin{aligned} E_{DD} &= E\left[(\tau-T)^+\right] = E[\tau-T|\tau\ge T]P(\tau\ge T) \\ &= E[\tau-T|\tau\ge T](1-P_{FA}) \approx E[\tau-T|\tau\ge T], \end{aligned} \tag{18}$$

for small P_{FA}. In [2], the E_{DD} is obtained via Monte Carlo simulation of $E[\tau-T|\tau\ge T]$. In this paper we obtain asymptotic estimates of $E[\tau-T|\tau\ge T]$. Of course since we have computed P_{FA}, then we can also obtain $E\left[(\tau-T)^+\right]$.

When $\mu = EZ_k > 0$, the time τ_γ for W_k at a local node to cross threshold γ satisfies, by SLLN, $\tau_x/x \to 1/\mu$ *a.s.* as $x \to \infty$. Thus for large γ, $\tau_\gamma \sim \gamma/\mu$.

Suppose there is only one local node, i.e., $L=1$. Then, after approximately τ_γ slots the local node will start transmitting signal level b to the fusion center. Let μ_l be the drift of fusion CUSUM F_k when l local nodes are transmitting. Hence, after τ_γ slots the drift of F_k is μ_1. Since $L=1$, μ_1 has to be positive. Then, the time for fusion center to touch threshold β, for large β is approximately β/μ_1. Therefore a reasonable asymptotic estimate of E_{DD}, for large γ and β, would be, $E_{DD} \approx \gamma/\mu + \beta/\mu_1$. We have seen that this gives a good match with the simulations even for small positive drifts μ, μ_1.

For $L \geq 2$, $\gamma/\mu + \beta/\mu_1$ is not a good approximation for E_{DD}. This is because of three reasons. First, when there is more than one node running CUSUM W_k, any one of them can cross γ, and the time for the first among them to cross is much less than $\frac{\gamma}{\mu}$, especially when L is large. Second, as the number of nodes crossing γ increases, the drift at the fusion node changes from μ_0 through μ_L. Finally, depending on the choice of I or D (based on whether (3) or (5) is used at the fusion center), some of the μ_l's can be negative or zero. Taking these factors into account, we have developed an approximation for E_{DD} which works quite well for $L > 1$. The details are omitted due to lack of space but will be provided in a detailed version of the paper.

Table II gives the comparison of our approximation computed above with simulated values of E_{DD} for various distributions. It can be seen that the approximation is good for all the cases considered here.

γ	$E[\tau_\gamma]$ Gauss	$E[\tau_\gamma]$ Pareto
5	930	800
6	2551	1100
7	6950	1455
8	19020	1880

TABLE I
MEAN FPT ($E[\tau_\gamma]$) FOR PARETO ($K=2.1$) AND GAUSSIAN WITH $EZ_k = -0.5$

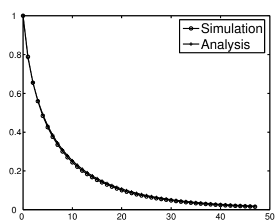

Fig. 6. Complementary CDF of Batch η for Laplace Z_k with $EZ_k = -0.3$ and $var(Z_k) = 1$ and $\gamma \geq 7$.

	L	I	γ	β	ρ	EZ_k	P_{FA} Anal	P_{FA} Sim	E_{DD} Anal	E_{DD} Sim
Gauss	2	1	15	18	0.005	-0.3	0.98e-4	1.09e-4	58.69	59.96
	5	2	15	18	0.005	-0.3	1.224e-4	1.1e-4	42.48	41
	10	2	15	18	0.005	-0.3	2.43e-4	2.28e-4	29.72	32
Laplace	2	1	16	16	0.005	-0.3	1.59e-4	1.6e-4	60.52	61
	6	2	16	16	0.005	-0.3	2.57e-4	2.06e-4	41.81	40
	12	3	16	16	0.005	-0.3	6.65e-5	5.5e-5	29.48	33.6
Lognormal	2	1	20	20	0.005	-0.3	8.98e-4	8.8e-4	75.9	78.5
	5	2	25	20	0.005	-0.3	1.47e-4	1.76e-4	74.06	71.7
	10	2	25	20	0.005	-0.3	2.97e-4	3.5-4	60.6	59.1
Pareto $K=50$	2	1	20	25	0.005	-0.3	1.28e-4	0.99e-4	79.24	83.52
Pareto $K=10$	2	1	10	12	0.005	-1	5.12e-4	6.77e-4	16.71	17.48
Pareto $K=2.1$	2	1	30	35	0.0005	-2	4.2e-3	4.65e-3	37.74	37.1

TABLE II
COMPARISON OF P_{FA} FOR VARIOUS DISTRIBUTIONS USING (5) AT THE LOCAL NODE (3) AT THE FUSION NODE. FOR E_{DD} WE HAVE USED $b=1$.

L	γ	β	P_{FA} Anal.	P_{FA} Sim.
2	20	20	3.44e-4	3.03e-4
	30	30	2.306e-5	2.28e-5
3	30	35	5.23e-5	4.7e-5
	40	45	1.658e-5	1.72e-5
5	30	30	1.928e-4	1.77e-4
	50	50	2.33e-5	2.52e-5

TABLE III
P_{FA} FOR PARETO $K=2.1$ ANALYSIS VS SIMULATION, FOR $EZ_k = -0.3$, $var(Z_k) = 1$, $I=1$ AND $b=1$.

IV. CONCLUSIONS

We have proposed a Page's CUSUM based energy efficient scheme in [2] which uses the physical layer fusion technique and CUSUM at the sensors as well as at the fusion center. In this paper we extend the algorithm to also include the nonparametric CUSUM. We have theoretically computed the probability of false alarm and mean of delay in change detection and compared with simulations.

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