International Symposium on Information Theory and its Applications, ISITA2008
Auckland, New Zealand, 7-10, December, 2008

A Joint Source-Channel Coding Scheme for Transmission of Correlated Discrete Sources over a Gaussian Multiple Access Channel

R Rajesh and Vinod Sharma

Dept. of Electrical Communication Engg.
Indian Institute of Science,Bangalore, India
Email: rajesh@pal.ece.iisc.ernet.in, vinod@ece.iisc.ernet.in

Abstract

We consider the problem of transmission of correlated discrete alphabet sources over a Gaussian Multiple Access Channel (GMAC). A distributed bit-to-Gaussian mapping is proposed which yields jointly Gaussian codewords. This can guarantee lossless transmission or lossy transmission with given distortions, if possible. The technique can be extended to the system with side information at the encoders and decoder.

Keywords: Gaussian multiple access channel, Joint source-channel coding, Discrete correlated sources.

1. INTRODUCTION AND SURVEY

Sensor networks are used in a wide variety of applications, one of the most common being the spatio-temporal monitoring of a random field [2]. In this case, the sensor nodes need to transmit their observations to a fusion center which uses this data to estimate the sensed random field. Since the sensor nodes has very limited energy and the transmission is very energy intensive, it is important to compress the transmitted data.

The sensor nodes transmit their observations to the fusion center (or a cluster head) usually over a Multiple Access Channel (MAC) ([3], [14]). Often the received symbol is a super-position of the transmitted symbols corrupted by additive white Gaussian noise (AWGN) yielding a Gaussian MAC (GMAC). Usually the observations sensed by the sensor nodes are discretized and then transmitted. This motivates us to consider the transmission of discrete alphabet sources over a GMAC. The physical proximity of the sensor nodes makes their observations correlated. This correlation can be exploited to compress the transmitted data. We propose a distributed 'correlation preserving' joint source-channel coding scheme yielding jointly Gaussian channel codewords which will be shown to compress the data efficiently. The proposed technique is interesting in other scenarios also.

In the following we survey the related literature. Cover, El Gamal and Salehi [4] provided sufficient conditions for transmitting losslessly discrete correlated observations over a discrete MAC. They also show that unlike for independent sources, the source-channel separation does not hold in this case. The correlation preserving mapping was extended to more general models in [1]. The results of [4] have been extended in [11] and [13] to the case of lossy transmission with side information.

In [8] it is shown that feed back increases the capacity of a GMAC. GMAC under received power constraints is studied in [5] and it is shown that the source-channel separation holds in this case. The GMAC with correlated jointly Gaussian sources is studied in [9] and the edge capacities obtained are similar to the expressions in [8]. In [7] one necessary and two sufficient conditions for transmission are provided for this case. The performance comparison of the schemes in [7] with a separation based scheme is given in [10].

This paper makes the following contributions. From our general results in [11] we obtain explicit conditions for transmission of correlated discrete sources over a GMAC. A new bit-to-Gaussian joint source-channel coding scheme is introduced to yield jointly Gaussian channel codewords for a GMAC. Examples are provided to show the efficiency of the scheme and the technique is extended to the case of lossy transmission with side information.

The paper is organized as follows. Sufficient conditions for transmission of discrete alphabet sources over a Gaussian MAC are given in Section 2. A new joint source-channel coding scheme is proposed in 3. An example is given in Section 4 to illustrate the results. In Section 5 the scheme is extended to lossy transmission and transmission with side information. Section 6 summarizes the results.

2. DISCRETE ALPHABET SOURCES OVER A GAUSSIAN MAC

We consider a system with two discrete alphabet sources

This work was partially supported by DRDO-IISc program on Advanced Research in Mathematical Engineering.

$\{U_{1k}, k \geq 1\}$ and $\{U_{2k}, k \geq 1\}$, each forming an iid sequence although (U_{1k}, U_{2k}) may be possibly dependent. Sequence $\{U_{ik}, k \geq 1\}, i = 1, 2$ is made available to encoder i only. The encoder i generates sequence $\{X_{ik}, k \geq 1\}$ to be transmitted over a memoryless MAC such that the receiver can reproduce $\{(U_{1k}, U_{2k}), k \geq 1\}$ losslessly, if possible. If MAC input at time k is $x_{ik}, i = 1, 2$ then its output Y_k will be distributed as $p(y|x_{1k}, x_{2k})$. By memoryless MAC we mean ($y^k \stackrel{\Delta}{=} (y_1, ..., y_k)$, similarly for other sequences) $p(y^n|x_1^n, x_2^n) = \prod_{k=1}^{n} p(y_k|x_{1k}, x_{2k})$. The MAC inputs X_{1k}, X_{2k} and output Y_k can take values in finite/countable/uncountable sets. In the following we will be interested in the GMAC where X_{1k}, X_{2k}, Y_k will be real valued.

At the end of the paper we will extend the model to also include multiple (≥ 2) sources, and the encoder and/or the decoder have certain side information. We will also allow lossy transmission.

Since in the above setup, our conditions for lossless transmission will have single letter characterization, instead of sequences $\{U_{1k}, k \geq 1\}$, $\{U_{2k}, k \geq 1\}$ we will only consider r.v.s (U_1, U_2) which have the distribution of (U_{1k}, U_{2k}). Similarly, (X_1, X_2, Y) will be r.v.s with the distribution of (X_{1k}, X_{2k}, Y_k) which are also iid sequences. Also, $X \leftrightarrow Y \leftrightarrow Z$ will indicate that $\{X, Y, Z\}$ forms a Markov chain.

Specializing the general results in [11] to the case of lossless transmission of two discrete sources (U_1, U_2) (generating iid sequences in time) over a general MAC, the sufficient conditions are

$$\begin{aligned} H(U_1|U_2) &< I(X_1; Y|X_2, U_2), \\ H(U_2|U_1) &< I(X_2; Y|X_1, U_1), \\ H(U_1, U_2) &< I(X_1, X_2; Y) \end{aligned} \tag{1}$$

where X_1, X_2 are the channel inputs, Y is the channel output and $X_1 \leftrightarrow U_1 \leftrightarrow U_2 \leftrightarrow X_2$ is satisfied. These are also the sufficient conditions available in [4] for discrete sources and channels.

In this section, we further specialize the above results for lossless transmission of discrete correlated sources over an additive memoryless Gaussian Multiple Access Channel (GMAC): $Y = X_1 + X_2 + N$ where N is a Gaussian random variable independent of X_1 and X_2. The noise N satisfies $E[N] = 0$ and $Var(N) = \sigma_N^2$. We will also have the transmit power constraints: $E[X_i^2] \leq P_i, i = 1, 2$. Since source-channel separation does not hold for this system, a joint source-channel coding scheme is needed for optimal performance.

The dependence of R.H.S. of (1) on input alphabets prevents us from getting a closed form expression for the admissibility criterion. Therefore we relax the conditions by taking away the dependence on the input alphabets. This will allow us to obtain good joint source-channel codes.

Lemma 1: Under our assumptions, $I(X_1; Y|X_2, U_2) \leq I(X_1; Y|X_2)$.

Proof: Let $\Delta \stackrel{\Delta}{=} I(X_1; Y|X_2, U_2) - I(X_1; Y|X_2)$. Then

$$\begin{aligned} \Delta &= h(Y|X_2, U_2) - h(Y|X_1, X_2, U_2) \\ &- [h(Y|X_2) - h(Y|X_1, X_2)]. \end{aligned}$$

where h is the differential entropy.

Since the channel is memoryless,

$$h(Y|X_1, X_2, U_2) = h(Y|X_1, X_2).$$

Thus, $\Delta = h(Y|X_2, U_2) - h(Y|X_2) \leq 0$. ■

Thus from (1),

$$\begin{aligned} H(U_1|U_2) &< I(X_1; Y|X_2, U_2) \leq I(X_1; Y|X_2), && (2) \\ H(U_2|U_1) &< I(X_2; Y|X_1, U_1) \leq I(X_2; Y|X_1), && (3) \\ H(U_1, U_2) &< I(X_1, X_2; Y). && (4) \end{aligned}$$

The relaxation of the upper bounds is only in (2), (3) and not in (4).

We show that the relaxed upper bounds are maximized if (X_1, X_2) is jointly Gaussian and the correlation ρ in X_1 and X_2 is high (the highest possible ρ may not give the largest upper bound in the three inequalities in (2)-(4)).

Lemma 2: A jointly Gaussian distribution for (X_1, X_2) maximizes $I(X_1; Y|X_2), I(X_2; Y|X_1)$ and $I(X_1, X_2; Y)$ simultaneously.

Proof: Since

$$\begin{aligned} I(X_1, X_2; Y) &= h(Y) - h(Y|X_1, X_2) \\ &= h(X_1 + X_2 + N) - h(N), \end{aligned}$$

it is maximized when $h(X_1 + X_2 + N)$ is maximized. This entropy is maximized when $X_1 + X_2$ is Gaussian with the largest possible variance $= P_1 + P_2$. If (X_1, X_2) is jointly Gaussian then so is $X_1 + X_2$.

Next consider $I(X_1; Y|X_2)$. This equals

$$\begin{aligned} h(Y|X_2) - h(N) &= h(X_1 + X_2 + N|X_2) - h(N) \\ &= h(X_1 + N|X_2) - h(N) \end{aligned}$$

which is maximized when $p(x_1|x_2)$ is Gaussian and this happens when X_1, X_2 are jointly Gaussian.

A similar result holds for $I(X_2; Y|X_1)$. ■

The difference between the bounds in (2) is

$$I(X_1, Y|X_2) - I(X_1, Y|X_2, U_2) = I(X_1 + N; U_2|X_2). \tag{5}$$

This difference is small if correlation between (U_1, U_2) is small. In that case $H(U_1|U_2)$ and $H(U_2|U_1)$ will be large and (2) and (3) can be active constraints. If correlation between (U_1, U_2) is large, $H(U_1|U_2)$ and $H(U_2|U_1)$ will be small and (4) will be the only active constraint. In this case the difference between the two bounds in (2) and (3) is large but not important. Thus, the outer bounds in (2) and (3) are close to the inner bounds whenever the constraints (2) and (3) are active. Often (4) will be the only active constraint.

An advantage of outer bounds in (2)-(4) is that we will be able to obtain a good source-channel coding scheme. Once (X_1, X_2) are obtained we can check for sufficient conditions (1). If these are not satisfied for the (X_1, X_2) obtained, we will increase the correlation ρ between (X_1, X_2) if possible (see details below). Increasing the correlation in (X_1, X_2) will decrease the difference in (5) and increase the possibility of satisfying (1) when the outer bounds in (2),(3) are satisfied.

We evaluate the (relaxed) rate region (2)-(4) for the Gaussian MAC with jointly Gaussian channel inputs (X_1, X_2) with the transmit power constraints. For maximization of this region we need mean vector $[0\ 0]$ and covariance matrix $K_{X_1,X_2} = \begin{pmatrix} P_1 & \rho\sqrt{P_1P_2} \\ \rho\sqrt{P_1P_2} & P_2 \end{pmatrix}$ where ρ is the correlation between X_1 and X_2. Then (2)-(4) provide the relaxed constraints

$$H(U_1|U_2) \quad < \quad 0.5\log\left[1 + \frac{P_1(1-\rho^2)}{{\sigma_N}^2}\right], \qquad (6)$$

$$H(U_2|U_1) \quad < \quad 0.5\log\left[1 + \frac{P_2(1-\rho^2)}{{\sigma_N}^2}\right], \qquad (7)$$

$$H(U_1, U_2) \quad < \quad 0.5\log\left[1 + \frac{P_1 + P_2 + 2\rho\sqrt{P_1P_2}}{{\sigma_N}^2}\right] \qquad (8)$$

Interestingly, these upper bounds are similar to those in [8] for the GMAC with feedback. In their set up the feedback induces correlation between the channel alphabets and the rate region is enlarged. The expressions are also same as the edge capacities of [9].

The upper bounds in the first two inequalities in (6) and (7) decrease as ρ increases. But the third upper bound (8) increases with ρ and often the third constraint is the limiting constraint.

This motivates us to consider the GMAC with correlated jointly Gaussian inputs. The next lemma provides an upper bound on the correlation between (X_1, X_2) in terms of the distribution of (U_1, U_2).

Lemma 3: Let (U_1, U_2) be the correlated sources and $X_1 \leftrightarrow U_1 \leftrightarrow U_2 \leftrightarrow X_2$ where X_1 and X_2 are jointly Gaussian. Then the correlation between (X_1, X_2) satisfies $\rho^2 \leq 1 - 2^{-2I(U_1,U_2)}$.

Proof: Since $X_1 \leftrightarrow U_1 \leftrightarrow U_2 \leftrightarrow X_2$ is a Markov chain, by data processing inequality $I(X_1; X_2) \leq I(U_1; U_2)$. Taking X_1, X_2 to be jointly Gaussian with zero mean, unit variance and correlation ρ, $I(X_1, X_2) = 0.5log_2(\frac{1}{1-\rho^2})$. This implies $\rho^2 \leq 1 - 2^{-2I(U_1,U_2)}$. ∎

It is stated in [12], without proof, that the correlation between (X_1, X_2) cannot be greater than the correlation of the source (U_1, U_2). Lemma 3 gives a tighter bound in many cases. Consider (U_1, U_2) with the joint distribution: $P(U_1 = 0; U_2 = 0) = P(U_1 = 1; U_2 = 1) = 1/3; P(U_1 = 1; U_2 = 0) = P(U_1 = 0; U_2 = 1) = 1/6$. The correlation between the sources is 0.33 but from Lemma 3, the correlation between (X_1, X_2) cannot exceed 0.327.

3. A CODING SCHEME

In this section we develop a coding scheme for mapping the discrete alphabets into jointly Gaussian correlated code words which also satisfy the Markov condition. The heart of the scheme is to approximate a jointly Gaussian distribution with the sum of product of Gaussian marginals. Although this is stated in the following lemma for two dimensional vectors (X_1, X_2), the results hold for any finite dimensional vectors.

Lemma 4: Any jointly Gaussian two dimensional density can be uniformly arbitrarily closely approximated by a weighted sum of product of marginal Gaussian densities:

$$\sum_{i=1}^{N} \frac{p_i}{\sqrt{2\pi c_{1i}}} e^{\frac{-1}{2c_{1i}}(x_1 - a_{1i})^2} \frac{q_i}{\sqrt{2\pi c_{2i}}} e^{\frac{-1}{2c_{2i}}(x_2 - a_{2i})^2}. \qquad (9)$$

Proof: By Stone-Weierstrass theorem ([6]) the class of functions $(x_1, x_2) \mapsto e^{\frac{-1}{2c_1}(x_1-a_1)^2} e^{\frac{-1}{2c_2}(x_2-a_2)^2}$ can be shown to be dense in C_0 under uniform convergence where C_0 is the set of all continuous functions on $\Re^2$ such that $\lim_{\|X\|\to\infty} |f(x)| = 0$. Since the jointly Gaussian density $(x_1, x_2) \mapsto e^{\frac{-1}{2\sigma^2}(\frac{x_1^2 + x_2^2 - 2\rho x_1 x_2}{1-\rho^2})}$ is in C_0, it can be approximated arbitrarily closely uniformly by the functions (9). ∎

From the above lemma we can form a sequence of functions $f_n(x_1, x_2)$ of type (9) such that $sup_{x_1,x_2}|f_n(x_1, x_2) - f(x_1, x_2)| \to 0$ as $n \to \infty$, where f is a given jointly Gaussian density. Although f_n are not guaranteed to be probability densities, due to uniform convergence, for large n, they will almost be. In the following lemma we will assume that we have made the minor modification to ensure that f_n is a proper density for large enough n. This lemma shows that obtaining (X_1, X_2) from such approximations can provide the (relaxed) upper bounds in (2)-(4) (we actually show for the third inequality only but this can be shown for the other inequalities in the same way). Let (X_{m1}, X_{m2}) and (X_1, X_2) be random variables with densities f_m and f and

$sup_{x_1,x_2}|f_m(x_1,x_2)-f(x_1,x_2)| \to 0$ as $m \to \infty$. Let Y_m and Y denote the corresponding channel outputs.

Lemma 5: For the random variables defined above, if $\{logf_m(Y_m), m \geq 1\}$ is uniformly integrable, $I(X_{m1}, X_{m2}; Y_m) \to I(X_1, X_2; Y)$ as $m \to \infty$.

Proof: Since

$$\begin{aligned} I(X_{m1}, X_{m2}; Y_m) &= h(Y_m) - h(Y_m|X_{m1}, X_{m2}) \\ &= h(Y_m) - h(N), \end{aligned}$$

it is sufficient to show that $h(Y_m) \to h(Y)$.

From $(X_{m1}, X_{m2}) \xrightarrow{d} (X_1, X_2)$ and independence of (X_{m1}, X_{m2}) from N, we get $Y_m = X_{m1} + X_{m2} + N \xrightarrow{d} X_1 + X_2 + N = Y$. Then $f_m \to f$ uniformly implies that $f_m(Y_m) \xrightarrow{d} f(Y)$. Since $f_m(Y_m) \geq 0$, $f(Y) \geq 0$ $a.s$ and log is continuous except at 0, we obtain $logf_m(Y_m) \xrightarrow{d} logf(Y)$. Then uniform integrability provides $I(X_{m1}, X_{m2}; Y_m) \to I(X_1, X_2; Y)$. ■

A set of sufficient conditions for uniform integrability of $\{logf_m(Y_m), m \geq 1\}$ is

(1) Number of components in (9) is upper bounded.

(2) Variance of component densities in (9) is upper bounded and lower bounded away from zero.

(3) The means of the component densities in (9) are in a bounded set.

From Lemma 4 any function in C_0 (in particular joint Gaussian density with any correlation) can be expressed by a linear combination of marginal Gaussian densities. But the coefficients p_i and q_i in (9) may be positive or negative. To realize our coding scheme, we would like to have the p_i's and q_i's to be non negative. This introduces constraints on the realizable Gaussian densities in our coding scheme. For example, from Lemma 3, the correlation ρ between X_1 and X_2 cannot exceed $\sqrt{1-2^{-2I(U_1;U_2)}}$. Also there is still the question of getting a good linear combination of marginal densities to obtain the joint density for a given N in (9).

This motivates us to consider an optimization procedure for finding $p_i, q_i, a_{1i}, a_{2i}, c_{1i}$ and c_{2i} in (9) that provides the best approximation to a given joint Gaussian density. We illustrate this with an example. Consider U_1, U_2 to be binary. Let $P(U_1 = 0; U_2 = 0) = p_{00}; P(U_1 = 0; U_2 = 1) = p_{01}; P(U_1 = 1; U_2 = 0) = p_{10}$ and $P(U_1 = 1; U_2 = 1) = p_{11}$. We can consider

$$\begin{aligned} f(X_1 = .|U_1 = 0) &= p_{101}\mathcal{N}(a_{101}, c_{101}) + p_{102}\mathcal{N}(a_{102}, c_{102}) \\ &\quad ... + p_{10r_1}\mathcal{N}(a_{10r_1}, c_{10r_1}), \qquad (10) \\ f(X_1 = .|U_1 = 1) &= p_{111}\mathcal{N}(a_{111}, c_{111}) + p_{112}\mathcal{N}(a_{112}, c_{112}) \\ &\quad ... + p_{11r_2}\mathcal{N}(a_{11r_2}, c_{11r_2}), \qquad (11) \\ f(X_2 = .|U_2 = 0) &= p_{201}\mathcal{N}(a_{201}, c_{201}) + p_{202}\mathcal{N}(a_{202}, c_{202}) \\ &\quad ... + p_{20r_3}\mathcal{N}(a_{20r_3}, c_{20r_3}), \qquad (12) \\ f(X_2 = .|U_2 = 1) &= p_{211}\mathcal{N}(a_{211}, c_{211}) + p_{212}\mathcal{N}(a_{212}, c_{212}) \\ &\quad ... + p_{21r_4}\mathcal{N}(a_{21r_4}, c_{21r_4}). \qquad (13) \end{aligned}$$

where $\mathcal{N}(a, b)$ denotes Gaussian density with mean a and variance b. Let $\underline{p}$ be the vector with components $p_{101}, ..., p_{10r_1}, p_{111}, ..., p_{11r_2}, p_{201}, ..., p_{20r_3}, p_{211}, ..., p_{21r_4}$. Similarly we denote by $\underline{a}$ and $\underline{c}$ the vectors with components $a_{101}, ..., a_{10r_1}, a_{111}, ..., a_{11r_2}, a_{201}, ..., a_{20r_3}, a_{211}, ..., a_{21r_4}$ and $c_{101}, ..., c_{10r_1}, c_{111}, ..., c_{11r_2}, c_{201}, ..., c_{20r_3}, c_{211}, ..., c_{21r_4}$.

Let $f_\rho(x_1, x_2)$ be the jointly Gaussian density that we want to approximate. Let it has zero mean and covariance matrix $K_{X_1,X_2} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$. Let $g_{\underline{p},\underline{a},\underline{c}}$ be the sum of marginal densities with parameters $\underline{p}, \underline{a}, \underline{c}$ approximating f_ρ . The best g is obtained by solving the following minimization problem:

$$\min_{\underline{p},\underline{a},\underline{c}} \int [g_{\underline{p},\underline{a},\underline{c}}(x_1, x_2) - f_\rho(x_1, x_2)]^2 dx_1 dx_2 \qquad (14)$$

subject to

$$(p_{00} + p_{01}) \sum_{i=1}^{r_1} p_{10i}a_{10i} + (p_{10} + p_{11}) \sum_{i=1}^{r_2} p_{11i}a_{11i} = 0,$$

$$(p_{00} + p_{10}) \sum_{i=1}^{r_3} p_{20i}a_{20i} + (p_{01} + p_{11}) \sum_{i=1}^{r_4} p_{21i}a_{21i} = 0,$$

$$\begin{aligned} &(p_{00} + p_{01}) \sum_{i=1}^{r_1} p_{10i}(c_{10i} + a_{10i}^2) + \\ &(p_{10} + p_{11}) \sum_{i=1}^{r_2} p_{11i}(c_{11i} + a_{11i}^2) = 1, \end{aligned}$$

$$\begin{aligned} &(p_{00} + p_{10}) \sum_{i=1}^{r_3} p_{20i}(c_{20i} + a_{20i}^2) + \\ &(p_{01} + p_{11}) \sum_{i=1}^{r_4} p_{21i}(c_{21i} + a_{21i}^2) = 1, \end{aligned}$$

$$\sum_{i=1}^{r_1} p_{10i} = 1, \sum_{i=1}^{r_2} p_{11i} = 1, \sum_{i=1}^{r_3} p_{20i} = 1, \sum_{i=1}^{r_4} p_{21i} = 1,$$

$$p_{10i} \geq 0, c_{10i} \geq 0 \; for \; i \in \{1, 2...r_1\},$$
$$p_{11i} \geq 0, c_{11i} \geq 0 \; for \; i \in \{1, 2...r_2\},$$
$$p_{20i} \geq 0, c_{20i} \geq 0 \; for \; i \in \{1, 2...r_3\},$$
$$p_{21i} \geq 0, c_{21i} \geq 0 \; for \; i \in \{1, 2...r_4\}.$$

The above constraints are such that the resulting distribution g for (X_1, X_2) will satisfy $E[X_i] = 0$ and $E[X_i^2] = 1, \; i = 1, 2$.

The above coding scheme will be used to obtain a codebook as follows. If user 1 produces $U_1 = 0$, then with probability p_{10i} the encoder 1 obtains codeword X_1 from the distribution $\mathcal{N}(a_{10i}, c_{10i})$. Similarly we obtain the codewords for $U_1 = 1$ and for user 2. Once we have found the encoder maps the encoding and decoding are as described in the proof of Theorem 1 in [11] . The decoding is done by joint typicality of the received Y^n with (U_1^n, U_2^n).

This coding scheme can be extended to any discrete alphabet case. We give an example below to illustrate the coding scheme.

4. EXAMPLE

Consider (U_1, U_2) with the joint distribution: $P(U_1 = 0; U_2 = 0) = P(U_1 = 1; U_2 = 1) = P(U_1 = 0; U_2 = 1) = 1/3; P(U_1 = 1; U_2 = 0) = 0$ and power constraints $P_1 = 3; P_2 = 4$. Also consider a Gaussian multiple access channel with $\sigma_N^2 = 1$. If the sources are mapped into independent channel code words, then the sum rate condition in (8) with $\rho = 0$ should hold. The LHS evaluates to 1.585 bits whereas the RHS is 1.5 bits. Thus condition (8) is violated and hence the sufficient conditions in (1) are also violated.

In the following we explore the possibility of using correlated (X_1, X_2) to see if we can transmit this source on the given MAC. The inputs (U_1, U_2) can be distributedly mapped to jointly Gaussian channel code words (X_1, X_2) by the technique mentioned above. The maximum ρ which satisfies (6) and (7) are 0.7024 and 0.7874 respectively and the minimum ρ which satisfies (8) is 0.144. Thus, we can pick a ρ which satisfies (6)-(8). From Lemma 3, ρ is upper bounded by 0.546. Therefore we want to obtain jointly Gaussian (X_1, X_2) satisfying $X_1 \leftrightarrow U_1 \leftrightarrow U_2 \leftrightarrow X_2$ with correlation $\rho \in [0.144, 0.546]$. If we pick a ρ that satisfies the original bounds, then we will be able to transmit the sources (U_1, U_2) reliably on this MAC. Without loss of generality the jointly Gaussian channel inputs required are chosen with mean vector $[0 \; 0]$ and covariance matrix $K_{X_1, X_2} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$. The ρ chosen is 0.3 and hence is such that it meets all the conditions (6)-(8). Also, we choose $r_1 = r_2 = r_3 = r_4 = 2$. We solve the optimization problem (14) via MATLAB to get the function g. The normalized minimum distortion, defined as $\int [g_{\underline{p}, \underline{a}, \underline{c}}(x_1, x_2) - f_\rho(x_1, x_2)]^2 dx_1 dx_2 / \int f_\rho^2(x_1, x_2) dx_1 dx_2$ is 0.137% when the marginals are chosen as:

$$f(X_1|U_1 = 0) = \mathcal{N}(-0.0002, 0.9108),$$
$$f(X_1|U_1 = 1) = \mathcal{N}(-0.0001, 1.0446),$$
$$f(X_2|U_2 = 0) = \mathcal{N}(-0.0021, 1.1358),$$
$$f(X_2|U_2 = 1) = \mathcal{N}(-0.0042, 0.7283).$$

The approximation is shown in Figure 1. If we take $\rho = 0.6$ which violates Lemma 3 then the approximation is shown in Fig 2. We can see from Fig 2 that the error in this case is more. Now the normalized marginal distortion is 10.5 %. The original upper bound in (2) and (3) for this example with $\rho = 0.3$ is $I(X_1; Y|X_2, U_2) = 0.792$, $I(X_2; Y|X_1, U_1) = 0.996$. Also, $I(X_1; Y|X_2) = 0.949$, $I(X_2; Y|X_1) = 1.107$. $H(U_1|U_2) = H(U_2|U_1) = 0.66$ and we conclude that the original bounds too are satisfied by the choice of $\rho = 0.3$.

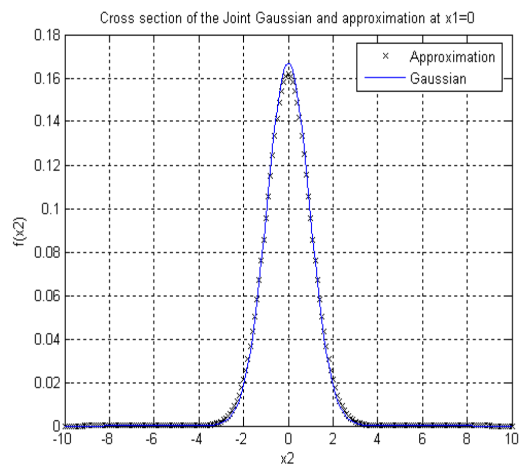

Figure 1: Cross section of the approximation of the joint Gaussian with $\rho = 0.3$

5. EXTENSIONS

The procedure mentioned in Section 3 can be extended to systems with discrete alphabets, multiple sources, lossy transmissions and side information. Consider multiple users with each source taking values in a discrete alphabet. Let N be the number of sources. Let user i generate symbols taking values in alphabets $\mathcal{U}_i$. In such a case for each user we find $P(X_i = .|U_i = u_i), \; u_i \in \mathcal{U}_i$ using the mapping mentioned in Section III to yield jointly Gaussian $(X_1, X_2, ..., X_N)$.

We can also consider distortion in the set-up by appropriate discrete auxiliary random variables W_i satisfying the distortion criterion and a suitable decoder. Consider the Example given in Section IV. Suppose there was no ρ that satisfied all the inequalities in (6)-(8). Then one could consider

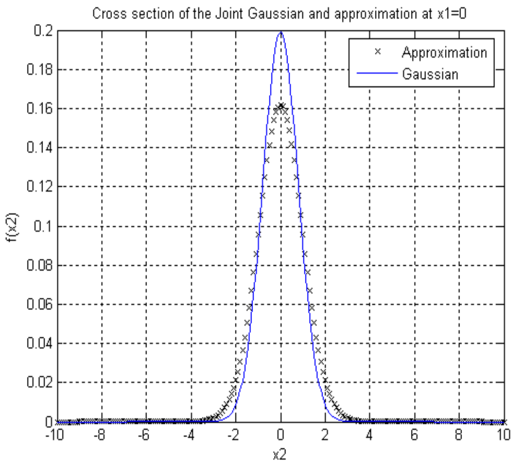

Figure 2: Cross section of the approximation of the joint Gaussian with $\rho = 0.6$

the transmission of (U_1, U_2) within a permissible distortion of (say) $d\%$. We can form auxiliary random variables W_1 and W_2 such that $X_1 \leftrightarrow W_1 \leftrightarrow U_1 \leftrightarrow U_2 \leftrightarrow W_2 \leftrightarrow X_2$ and we can recover (U_1, U_2) from (W_1, W_2) within the permissible distortion. We need to choose (W_1, W_2) such that the generalization of (6)-(8) to lossy case given in [11] are satisfied, if possible. Then we can form (X_1, X_2) from (W_1, W_2) via the optimization procedure(14).

The same procedure of finding the auxiliary random variables holds for the cases with side information also. For example if Z_1 and Z_2 are the side information available, then $f(X_i = .|U_i, Z_i),\ i \in \{1, 2\}$ can be obtained via the optimization procedure (14).

6. CONCLUSIONS

We have provided a bit-to-Gaussian mapping which provides jointly Gaussian codewords in a distributed fashion. This technique is used for the transmission of correlated discrete sources over a GMAC. It is also extended to multiple sources, discrete alphabets, lossy transmission and cases with side information.

References

[1] R. Ahlswede and T. Han, "On source coding with side information via a multiple access channel and related problems in information theory," *IEEE Trans. Inform. Theory*, vol. IT-29, no. 3, pp. 396–411, 1983.

[2] I. F. Akylidiz, W. Su, Y. Sankarasubramaniam, and E. Cayirici, "A survey on sensor networks," *IEEE Communications Magazine*, pp. 1–13, August 2002.

[3] S. J. Baek, G. Veciana, and X. Su, "Minimizing energy consumption in large-scale sensor networks through distributed data compression and hierarchical aggregation," *IEEE JSAC*, vol. 22, no. 6, pp. 1130–1140, Aug 2004.

[4] T. M. Cover, A. E. Gamal, and M. Salehi, "Multiple access channels with arbitrarily correlated sources," *IEEE Trans. Inform. Theory*, vol. IT -26, pp. 648–657, 1980.

[5] M. Gastpar, "Multiple access channels under received-power constraints," *Proc. IEEE Inform. Theory Workshop*, pp. 452–457, 2004.

[6] J. Jacod and P. Protter, *Probability Essentials*. Springer, N.Y., 2004.

[7] A. Lapidoth and S. Tinguely, "Sending a bi- variate Gaussian source over a Gaussian MAC," *IEEE ISIT 06*, 2006.

[8] L. H. Ozarow, "The capacity of the white Gaussian multiple access channel with feedback," *IEEE Trans. Inform. Theory*, vol. IT -30, pp. 623 – 629, 1984.

[9] S. S. Pradhan, S. Choi, and K. Ramchandran, "A graph based framework for transmission of correlated sources over multiple access channels," *http://arxiv.org/pdf/cs.IT/0608006*, march 2005.

[10] R. Rajesh and V. Sharma, "Source channel coding for Gaussian sources over a Gaussian multiple access channel," *Proc. 45 Allerton conference on Computing Control and Communication*, 2007.

[11] R. Rajesh, V. K. Varshneya, and V. Sharma, "Distributed joint source-channel coding on a multiple access channel with side information," *In proc. IEEE ISIT 2008, Toronto, Canada*.

[12] S. Ray, M. Medard, M. Effros, and R. Kotter, "On separation for multiple access channels," *Proc. IEEE Inform. Theory Workshop*, 2006.

[13] V. K. Varsheneya and V. Sharma, "Distributed coding for multiple access communication with side information," *Proc. IEEE Wireless Communication and Networking Conference (WCNC)*, April 2006.

[14] V. V. Veeravalli, "Decentralized quickest change detection," *IEEE Trans. Inform. Theory*, vol. IT-47, pp. 1657–65, 2001.