Cyclic Prefix Based Cooperative Sequential Spectrum Sensing Algorithms for OFDM

ArunKumar Jayaprakasam, Vinod Sharma, Chandra R. Murthy and Prashant Narayanan
Electrical Communication Engineering Department,
Indian Institute of Science, Bangalore 560012, India
Email: arunkumar.jayaprakash@yahoo.in, vinod@ece.iisc.ernet.in, cmurthy@ece.iisc.ernet.in, prashant2413@gmail.com

Abstract— **This paper considers the problem of spectrum sensing in cognitive radio networks when the primary user is using Orthogonal Frequency Division Multiplexing (OFDM). For this we develop cooperative sequential detection algorithms that use the autocorrelation property of cyclic prefix (CP) used in OFDM systems. We study the effect of timing and frequency offset, IQ-imbalance and uncertainty in noise and transmit power. We also modify the detector to mitigate the effects of these impairments. The performance of the proposed algorithms is studied via simulations. We show that sequential detection can significantly improve the performance over a fixed sample size detector.**

Keywords- Cognitive Radio, Cooperative Spectrum Sensing, Decentralized Sequential Detection, OFDM.

I. INTRODUCTION

Cognitive Radios, also called the secondary users, use the radio spectrum licensed to other (primary) users. They perform radio environment analysis, identify the spectral holes and then operate in those holes [1]. *Spectrum Sensing (SS)*, an essential first step in enabling Cognitive Radio (CR) technology involves, 1) identifying spectrum opportunities by detecting holes (white space) when they become available and 2) detecting when the primary reclaims an identified spectral hole. This needs to be done such that the guaranteed maximum interference levels to the primary are not exceeded and there is efficient use of spectrum by the secondary. Thus, the CR needs to detect reliably, quickly and robustly, possibly weak primary user signals. For example the IEEE 802.22 standard [1] requires a sensitivity of -116dBm, while keeping the probability of miss detection under 0.1, using a sensing duration < 2 seconds. Orthogonal Frequency Division Multiplexing (OFDM) is used in 802.11a/g wireless LAN's (WLAN), Wireless MAN's (IEEE 802.16 WiMAX,) 3GPP Long Term Evolution (LTE), etc. Because of its widespread acceptance and deployment, it is likely that a primary user would be using OFDM, thus making the problem of detecting OFDM signals especially relevant for cognitive radio. Most of the OFDM systems also employ a cyclic prefix (CP) which implies that the autocorrelelation is non-zero at delays of the useful symbol length – a property that can be exploited for SS [2]. Thus in this paper we focus on the problem of SS when the primary is using OFDM. Literature on SS is vast despite it being a relatively recent topic of research. We now briefly summarize the relevant recent work.

A. Literature Survey

For spectrum sensing, primarily three signal processing techniques are proposed in literature: Matched filter [3], Energy Detection [3] and cyclo-stationary feature detection [4]. Matched filtering is optimal but requires detailed knowledge of primary signaling. When no such knowledge is available an Energy Detector is optimal [3]. Hence most of the literature is based on energy detection. However unlike for the matched-filter and the cyclo-stationary detectors, it suffers from SNR wall problem in the presence of transmit power or receiver noise power uncertainties [3].

Cooperative SS where the decisions of different secondaries are fused to obtain the final decision has been studied in [5] and [6]. It is shown that cooperation can help mitigate the effects of fading, shadowing and hidden node problem. Sequential detection techniques have been used in [7], [8], [9] and [10]. These have been shown to outperform the snapshot (fixed samples size) detectors often used in SS.

SS in OFDM environment has been studied in [2], [11] and for OFDMA systems in [12]. In [11] CP correlation based snapshot and sequential detectors are studied. In [12] the effect of time and frequency offset on CP based snapshot detector is studied. In [13], the joint sensing and channel scheduling problem is addressed for OFDM-based CR systems.

B. Our Contribution

In this paper, we provide SS algorithms (for detecting spectral holes in time) when the primary is using OFDM. As in recent work on SS in OFDM, we exploit the autocorrelation property in our SS algorithms. Also, we study how some of the common impairments in a secondary receiver ([14]), e.g., frequency offset, timing offset, IQ-imbalance ([15]) and uncertainty in the noise power and unknown channel gains affect the correlation statistics, and thus the performance of the CP-detector. We propose techniques to modify the CP detector to mitigate some of the losses. These techniques are presented in the context of fixed sample-size detectors, but are extended to the sequential detection setup as well. Here, we are primarily interested in the sequential detection algorithms which are more efficient than fixed sample size (snapshot) detectors.

To alleviate the shadowing and hidden node problem, we propose cooperative sequential SS algorithms. In particular, we use DualCUSUM, a distributed, cooperative sequential-detection algorithm developed in [16] for detection of the switching ON or OFF of a primary using OFDM based on the CP correlation feature of the OFDM signal. In [10] DualCUSUM was shown to be an efficient SS algorithm for energy detectors.

This paper is organized as follows. Section II describes the OFDM model. We also present our cooperative sequential detection based setup. In Sec.III we study the effect of the impairments on the snapshot CP-detector and present possible techniques to overcome the same. In Sec.IV we extend these techniques to the sequential change detection algorithms. Section V concludes the paper.

II. MODEL

We consider a CR network which is sensing a primary using an OFDM system. The OFDM system of the primary consists of L_d narrowband signals $D_0, D_1, ..., D_{L_d-1}$ carried by the subcarriers. We will assume that $\{D_n, n \geq 1\}$ are independent identically distributed (i.i.d) with zero mean and finite variance. The OFDM symbol is obtained by passing the L_d signals through an inverse fast Fourier transform (IFFT). In addition, a cyclic prefix of length L_c is also appended to make the total OFDM symbol duration $L_s = L_d + L_c$. The inter-carrier spacing is denoted Δf.

Let, $d(k) = \sum_{n=0}^{L_s-1} D_n \exp(j2\pi kn / L_d);\ k = 0,1,..L_d - 1$. The baseband OFDM signal at time k is,

$$\begin{aligned} S(k) &= d(k + L_d - L_c),\ k = 0,1,..., L_c - 1, \\ S(k) &= d(k - L_c),\quad k = L_c, ..., L_s - 1. \end{aligned} \tag{1}$$

By Central Limit Theorem (CLT), $S(k)$ is approximately Gaussian. Also as $E[D_n] = 0$, $E[S(k)] = 0$. The parameters $L_d, L_c, \Delta f$ are assumed to be known to the secondary systems. In (1) we observe that the symbols $S(0),..., S(L_c - 1)$ are repeated as $S(L_s - L_c),..., S(L_s - 1)$. Thus, if we correlate this sequence with a shift of L_d, we will get a good correlation in case the primary is transmitting; otherwise not. CP correlators exploit this property in detecting the primary signal.

We consider a cognitive radio system with L secondary users that sense a channel via CP-Detectors. The observations made on the channel by these secondaries are processed and sent to a fusion center, which makes a decision on whether the channel is free or not. Then that decision is sent to all the secondary users for possible use of the channel. The secondary system has to detect when the primary starts transmission (OFF→ON) and when it stops (ON→OFF). In the following, we explain our setup for OFF→ON, but our algorithms work for ON→OFF also.

Let the primary start transmission at a random time T. Then at time k the signal received by the l^{th} secondary is,

$$\begin{aligned} X(k,l) &= N(k,l), \quad k = 1, 2, ..., T - 1, \\ X(k,l) &= h(l)S(k) + N(k,l),\ k = T, T+1, ... \end{aligned} \tag{2}$$

where $h(l)$ is the channel gain of the l^{th} user, and $N(k,l)$ is observation noise at the l^{th} user. We assume that the fading is frequency flat and remains constant during the interval of the observation (say, approximately, for a duration of ON/OFF period). Slow fading scenarios with primary staying ON and OFF for a few seconds will approximately satisfy this. This assumption is commonly made in literature [5], [8]. We also assume that $\{S(k), k \geq 1\}$ and $\{N(k), k \geq 1\}$ are i.i.d. sequences independent of each other and T. Thus, the pre-change distribution of $X(k,l)$ is $N_c(0, \sigma^2_{w,l})$, where N_c denotes the complex-normal distribution and $\sigma^2_{w,l}$ the noise power at node l. The post-change distribution of $X(k,l)$ is $N_c(0, \sigma^2_{w,l} + \sigma^2_{s,l})$ where $\sigma^2_{s,l}$ is the received power of the primary node at node l. The effect of different channel gains is absorbed into $\sigma^2_{s,l}$.

The aim is to detect the change (at random time T) at the fusion center as soon as possible at a time $\tau\ (\geq T)$ (i.e., to minimize $E[(\tau - T)^+]$, where $(x)^+ = x$ if $x \geq 0$ and 0 otherwise) using the messages transmitted from the L secondaries with an upper bound on probability of false alarm, $P_{FA} = P(\tau < T) \leq \alpha$. For this, each of the L nodes uses its observation $X(k,l)$ to generate a signal $Y(k,l)$ and transmits to the fusion center. The data received at the fusion center is corrupted by the Additive White Gaussian noise (AWGN) $Z(k)$ at the receiver. The fusion center uses the observations $Y(k,1),..., Y(k,L)$ to decide between the two hypotheses H_0 and H_1. If H_0 is chosen, the secondaries continue to use the channel in slot k and the SS session continues (they may use part of each slot for sensing and the rest for transmission). If H_1 is detected, the secondaries typically switch over to an alternate channel. To transmit $Y(k,1),..., Y(k,L)$ from the L secondaries to their fusion node, they need a Multiple Access Control (MAC) protocol. Time Division Multiple Access (TDMA) is the most commonly used protocol.

We have developed (see [7], [16]) a robust cooperative algorithm for SS in this scenario. We study this algorithm in the OFDM setup. We also modify it to take care of the various impairments commonly encountered in OFDM systems.

III. CYCLIC PREFIX BASED DETECTOR

In this section we explain the CP correlation based detector in the context of a single secondary node (thus subscript l will be omitted in the notation) and present how we mitigate the effects of different impairments and uncertainties. Also to simplify the presentation, we will in this section ignore the sequential aspects of the detection problem and consider only the snapshot detection: Given a number of observations $X(1), ..., X(ML_S)$ from M slots of OFDM symbols, we want to detect if H_0 or H_1 is true. We use the Neymon-Pearson (NP) method for detection. We compute the autocorrelation

$$R = \frac{1}{ML_c} \sum_{j=0}^{M-1} \sum_{i=1}^{L_c} X(jL_s + i) X^*(jL_s + i + L_d) \tag{3}$$

where X^* is the complex conjugate of X. The above detector assumes perfect OFDM symbol level synchronization and thus correlates only the exact set of samples which would be repeated in the CP under H_1. Using the CLT it can be shown [2] that $R \sim N_c(0,\sigma_0^2)$ under H_0 and $R \sim N_c(\sigma_s^2,\sigma_1^2)$ under H_1, where $\sigma_0^2 = \sigma_w^4 / ML_c$ and $\sigma_1^2 = ((\sigma_w^2+\sigma_s^2)^2)/ML_c$. At low SNR (i.e., $\sigma_s^2 << \sigma_w^2$), $\sigma_0^2 \approx \sigma_1^2$. We work under this assumption, as we are interested in detection at low SNR.

Since the post-change mean is real, under low SNR conditions, detection is based on $R_r = real(R)$ as $R_r \sim N(0,\sigma^2)$ under H_0 and $R_r \sim N(\sigma_s^2,\sigma^2)$ under H_1, where $\sigma^2 \approx \sigma_w^4 / ML_c$. The detection rule is of the form $R_r > \lambda$ for declaration of H_1.

Next, we discuss different impairments and possible techniques to mitigate their effects.

A. Timing offset

Timing offset occurs because the cognitive receiver may not know where the OFDM symbol boundary starts in the received set of samples. Thus, it may not know the exact set of samples to correlate in (3). If it correlates at an incorrect position, $E[R] \approx 0$ under H_1. One possible way to take care of this is to correlate for the duration of the entire OFDM symbol:

$$R_r = \mathrm{Re}\left(\frac{1}{ML_s}\sum_{i=1}^{ML_s} X(i)X^*(i+L_d)\right). \qquad (4)$$

Now $\sigma^2 \approx \sigma_w^4/2ML_s$ under either hypothesis. The post change mean under H_1 is $\mu\sigma_s^2$, where $\mu = L_c/L_s$. Because of this, one can expect the performance to degrade. Using $L_d = 64$, $L_c = 16, \Delta f = 10MHz/64, M = 100$ and SNR = −10dB ($\sigma_w^2 = 20, \sigma_s^2 = 2$) we simulated this setup to show the effects of timing offset. We will use these parameters throughout this paper. The unknown timing offset is chosen as 10. For different values of the probability of false alarm p_{fa}, the detection probability p_d is shown in Table 1.

To regain some of the lost performance instead of correlating over the entire set of samples we can estimate the timing offset θ by a maximum likelihood estimator (MLE) [14] as:

$$\hat{\theta}_{ML} = \underset{\theta\in\{1,2,\ldots,L_s-1\}}{\arg\max}\{\mathrm{Re}(R(\theta)) - \omega P(\theta)\} \qquad (5)$$

$$R(\theta) = \sum_{j=0}^{M-1}\sum_{i=1}^{L_c} X(jL_s+i+\theta)X^*(jL_s+i+L_d+\theta),$$

$$P(\theta) = \frac{1}{2}\sum_{j=0}^{M-1}\sum_{i=1}^{L_c}(|X(jL_s+i+\theta)|^2 + |X^*(jL_s+i+L_d+\theta)|^2),$$

$$\omega = \sigma_s^2/(\sigma_s^2+\sigma_w^2).$$

Under low SNR, ω is small. Also, for a large number of OFDM symbols, $P(\theta) \approx ML_c(\sigma_s^2+\sigma_w^2)$ under H_1 and $P(\theta) \approx ML_c\sigma_w^2$ under H_0. Thus, $\omega P(\theta)$ does not affect the max operation in (5), and we use the simplified estimator

$$\hat{\theta}_{ML} = \underset{\theta}{\arg\max}\{\mathrm{Re}(R(\theta))\}. \qquad (6)$$

This estimator has the advantage of not requiring knowledge of σ_s^2 and σ_w^2. Then we use the decision statistic:

$$R_r = \mathrm{Re}\left(\frac{1}{ML}\sum_{j=0}^{M-1}\sum_{i=1}^{L_c} X(jL_s+i+\hat{\theta}_{ML})X^*(jL_s+i+L_d+\hat{\theta}_{ML})\right) \qquad (7)$$

instead of (3). Under H_0 and H_1, now R_r is no longer normally distributed, but the Gaussian distribution still provides a good fit: the empirical distribution of R_r under either hypothesis and the normal fit is shown in Figure 1. Thus we use this approximation for designing the detection threshold and performance analysis.

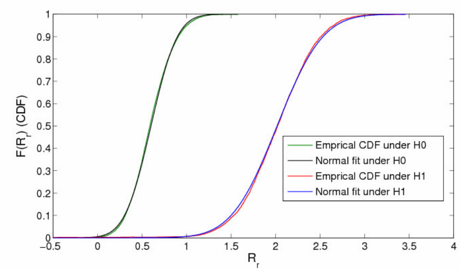

Figure 1: Empirical CDFs of R_r

However, as the variances under H_0 and H_1 are different, the optimal likelihood ratio is not a linear function of R_r and involves knowledge of σ_s^2 at the CR, which is not desirable. Thus, we propose to continue to use tests of the form $R_r > \lambda$ which is sub-optimal in this case, and could be viewed as a non parametric test. The performance comparison is shown in Table 1. One sees that we recover most of the lost performance due to timing offset.

p_{fa}	No Impairments (3)	Correlating over entire symbol (4)	Timing Offset estimate (7)
0.05	0.9999	0.7921	0.9975
0.025	0.9996	0.7010	0.9944
0.01	0.9988	0.5770	0.9880

Table 1: Effect of Timing Offset in p_d

B. Frequency offset

Let us now consider the scenario when only a frequency offset is present (i.e., the timing offset is assumed to be known). Let the frequency offset (between the cognitive receiver and the primary transmitter) be denoted by φ, normalized with respect to the carrier spacing Δf. The received signal can be written as $X(k) = S(k)\exp(j2\pi\varphi k/L_d) + N(k)$. Under H_1, $R \sim N_c(\sigma_s^2 e^{-j2\pi\varphi},\sigma_1^2)$. If the receiver is not aware of the frequency offset, then post change $R_r \sim N(\sigma_s^2\cos(2\pi\varphi),\sigma^2)$, degrading the performance. (see Table 2, for $\varphi = 0.1$.) To mitigate this effect, we estimate the frequency offset φ via MLE $\hat{\varphi}_{ML}$ when the likelihood ratio is

$$2\sigma_s^2(R_r\cos(2\pi\varphi)+R_I\sin(2\pi\varphi))/\sigma_1^2 \ . \quad (8)$$

It can be shown that $\hat{\varphi}_{ML} = -\angle R/2\pi$, and we use this estimate in the NP test. Thus the optimal test becomes $|R|^2 > \lambda'$. Under H_0, $|R|^2$ has an exponential distribution, and under H_1, it has a non-central chi-square distribution. The performance is shown in Table 2 via simulations for the parameters used in Section III-A. When both timing and frequency offset are present, one can estimate these in two stages as

$$\hat{\theta}_{ML} = \arg\max_{\theta}\{|R(\theta)|\};\hat{\varphi}_{ML} = -\frac{1}{2\pi}\angle R(\hat{\theta}_{ML}) \ . \quad (9)$$

We will use it in the following when we consider all impairments together.

C. IQ-Imbalance

IQ-imbalance occurs due to non-ideal front end components in the receiver ([15]) resulting in the amplitude and phase imbalance in the inphase (I) and quadrature (Q) components of the signal. In the presence of IQ-imbalance the actual received signal is written as

$X(k) = \alpha Y(k) + \beta Y^*(k)$

where $Y(k) = S(k) + N(k),\ \alpha = \cos(\Delta\phi) + j\varepsilon\sin(\Delta\phi);$

$\beta = \varepsilon\cos(\Delta\phi) - j\sin(\Delta\phi)$ and ε and $\Delta\phi$ are the amplitude and phase imbalance parameters respectively. It can be shown that in the presence of IQ-imbalance,

$$R_r \sim N(0,\sigma_{IQ}^2) \quad \text{under } H_0, \quad (10)$$

$$R_r \sim N((1+\varepsilon^2)\sigma_s^2,\sigma_{IQ}^2) \quad \text{under } H_1$$

where $\sigma_{IQ}^2 \approx \sigma_w^4((1+\varepsilon^2)^2 + 4|C_1|^2)/2ML_c$ under low SNR conditions and $C_1 = \alpha\beta^*$. The detector performance is shown in Table 2 for $\Delta\phi = 10^\circ;\varepsilon = 0.2$ (and the other parameters as in Section III-A). We see that the performance of the detector degrades slightly even when the knowledge of imbalance parameters are assumed but not compensated for. However, we can further improve performance by compensating for it. We use the algorithm in [15] to compensate for IQ-Imbalance before starting the CP-detector. The imbalance parameters are estimated and corrected for as follows. Let

$$\kappa^2 = \sum_i^{ML_s} X_r^2(i) / \sum_i^{ML_s} X_i^2(i) \text{ and } \hat{\varepsilon} = (\kappa-1)/(\kappa+1).$$

Then one can correct the amplitude imbalance by

$$Z_r(k) = X_r(k)/(1+\hat{\varepsilon}),\ Z_i(k) = X_i(k)/(1-\hat{\varepsilon}) \quad (11)$$

Assuming the phase imbalance $\in [-\pi/4, \pi/4]$, it is estimated and corrected as,

$$\delta = -\sum_{i=1}^{MT_x} X_r(i)X_i(i) / \sum_{i=1}^{MT_x}(X_r^2(i) + X_i^2(i)), \quad \Delta\hat{\phi} = \sin^{-1}(2\delta)/2.$$

Then, instead of using the observations $X(k)$, we use $X'(k)$ with real and imaginary components

$$\begin{bmatrix} X'_r(k) \\ X'_i(k) \end{bmatrix} = \begin{bmatrix} \cos(\Delta\hat{\phi}) & \sin(\Delta\hat{\phi}) \\ \sin(\Delta\hat{\phi}) & \cos(\Delta\hat{\phi}) \end{bmatrix} \begin{bmatrix} Z_r(k) \\ Z_i(k) \end{bmatrix} \quad (12)$$

for the CP detector. The performance of the detector with the above estimator is shown in Table 2. We see almost no performance loss. From these results, we see that performance loss due to IQ imbalance could be ignored. However, we have found that it does cause non-negligible degradation when there are other impairments mentioned above. Then the improvement resulting from (11), (12) can be more significant.

p_{fa}	Frequency Offset without compensation	Frequency Offset with compensation (9)	IQ-Imbal., No compensation (10)	IQ-Imbal., with compensation (11, 12)
0.05	0.9965	0.9989	0.9991	0.9999
0.025	0.9913	0.9975	0.9977	0.9996
0.01	0.9794	0.9939	0.9937	0.9988

Table 2: p_d under Frequency Offset and IQ-Imbalance

D. Noise/Transmit power uncertainty

In cognitive radio setting, often the receiver noise power σ_w^2 and the received signal power σ_s^2, may not be exactly known to the cognitive radio [3]. We address the detection problem under these uncertainties. Under noise uncertainty, since the variance of R_r is dependent on the noise power, the detection threshold cannot be set independently of it. Thus, the noise power is estimated as $\hat{\sigma}_w^2 = \text{var}(X) \approx \sum_{i=1}^{ML_s} X(i)X^*(i)/ML_s$, and this is used to set the threshold λ to achieve desired p_{fa}. This causes a minor performance loss, as under H_1 the threshold will be slightly higher as $\text{var}(X) \approx (\sigma_w^2 + \sigma_s^2)^2/ML_s$. But, as can be seen from Table 3, the performance degradation is negligible. Also, we have been using tests of the form $R_r > \lambda$ or $|R_r| > \lambda$ (partly motivated by the constraints of the present section), the statistics of which does not depend upon σ_s^2 under H_0, knowledge of receive signal power is not necessary to set the threshold λ to achieve the desired p_{fa}.

E. All impairments

In this section, we simulate the performance of the fixed sample size CP-detector when all impairments are present. First, the detector estimates and compensates for IQ imbalance using (12) and (13). Then, the variance of the received signal is estimated to set the threshold. Next, the optimal timing and frequency offsets are estimated using (9) and the test is of the form $|R| > \lambda$. The performance is shown in Table 3, under the impairments and data statistic given in this section. We see that our estimation schemes recover most of the losses.

For reference, we also compare with the detector in (4) in the presence of IQ-imbalance, frequency offset and noise uncertainty. Noise uncertainty for this detector is taken care as in III.D (i.e., estimating the noise variance to adjust the threshold) as this is necessary to set the threshold. It takes care of timing offset by correlating over the entire OFDM symbol duration, but the detector is unaware of frequency offset and IQ imbalance. Thus, even by partially compensating for the impairments, the performance can be very bad. However, from the last column in Table 3, we see that using our methods, most of the losses can be recovered.

Motivated by these results, we mitigate the effects of these impairments in DualCUSUM in the next section using similar techniques.

p_{fa}	Noise power estimation	All Impair. ((4) with noise power estimation)	All Impairments with all compensation
0.05	0.9999	0.5122	0.9712
0.025	0.9995	0.3908	0.9556
0.01	0.9976	0.2614	0.9300

Table 3: p_d under noise uncertainty and all impairments

IV. COOPERATIVE SEQUENTIAL SENSING OF OFDM

In this section, we apply cooperative sequential detection algorithms developed in [7], [10], [16] for sensing in the OFDM setup of Sec.II. Interested readers are referred to [7], [10], [16] for a more detailed introduction and its advantages (which we skip here due to lack of space). We study the performance of the cooperative algorithms with different levels of uncertainty. We also show the benefits of sequential detection algorithms vis-à-vis snapshot detection.

DualCUSUM uses the well known CUSUM algorithm [17] at the cognitive receivers as well as at the fusion node for detection of change (ON→OFF and OFF→ON for the primary). CUSUM is known to be optimal in different scenarios and uses log likelihood ratio (LLR). Consequently DualCUSUM has also been shown to perform very well [10], [16]. In the following we use DualCUSUM in our present scenario and use the estimation schemes discussed in Sec.III (suitably modified) to overcome the effects of different impairments.

A. Dual CUSUM with no impairments

We consider the ideal scenario where there are none of the impairments mentioned in Sec.III. Correlation is performed only over a length of samples corresponding to the cyclic prefix. Since all the parameters, including noise and received primary power are known, one can apply the DualCUSUM [16] as explained briefly below. Each node l computes the LLR $\xi_{j,l}$ of $R_r(j,l)$ in each slot j of L_s samples as

$$R_r(j,l) = \mathrm{Re}\left(\frac{1}{L_c}\sum_{i=1}^{L_c} X(jL_s+i,l)X^*(jL_s+i+L_d,l)\right) \quad (13)$$
$$\xi_{j,l} = (2\sigma_{s,l}^2 R_r(j,l) - \sigma_{s,l}^4)\big/(2\sigma_w^4/L_c)$$

and calculates the CUSUM $W_{j,l} = (W_{j-1,l}+\xi_{j,l})^+$. If the CUSUM crosses a threshold γ, it transmits a message $Y_{j,l} = b1_{\{W_{j,l}>\gamma\}}$ to the fusion node (i.e., it sends a '1' with amplitude b). The fusion center receives Y_j in slot j where, $Y_j = \sum_l Y_{j,l} + Z_j$. (Physical layer fusion is assumed at fusion node, which receives the information in presence of AWGN noise $Z_j \sim N(0,\sigma_M^2)$). The fusion node also runs CUSUM based on its input Y_j by using the LLR ζ_j as follows:

$$\zeta_j = \left(2Y_j bI - (bI)^2\right)\big/(2\sigma_M^2); \;\; F_k = \max\{0, F_{k-1}+\zeta_j\}; \quad (14)$$

and finally declares change at τ if F_k exceeds a threshold β, i.e., $\tau = \inf\{k : F_k > \beta\}$.

The parameters γ,β,b,I affect the performance of the algorithm and the techniques developed in [16] can be used to optimize these parameters.

B. DualCUSUM when timing offset is unknown

For the timing offset case, the decision statistic used at each node is as follows. As the timing offset is unknown, we cannot use the detector in (3). Also, the timing offset estimator of (7) is not preferred, as under low SNR conditions, to minimize the estimation error, we need a large number of OFDM symbols [14]. In the sequential setup we can improve over (7) by using an iterative estimator.

Thus we propose the following. Each node runs L_s CUSUMs for each possible timing offset of the primary. In each slot j, each node l computes

$$\underset{m\in\{0,1,..L_d-1\}}{R_r(j,l,m)} = \mathrm{Re}\left(\frac{1}{L_c}\sum_{i=1}^{L_c} X(jL_s+i+m,l)X^*(jL_s+i+m+L_d,l)\right),$$
$$\xi_{j,l,m} = (2\sigma_{s,l}^2 R_r(j,l,m) - \sigma_{s,l}^4)\big/(2\sigma_w^4/L_c);\; W_{j,l,m} = (W_{j-1,l,m}+\xi_{j,l,m})^+; \quad (15)$$
$$W_{j,l} = \max_{m\in\{0,1,..L_d-1\}} W_{j,l,m};\; Y_{j,l} = b1_{\{W_{j,l}>\gamma\}}.$$

Since none of the impairments at the secondary nodes has any effect at the statistics of observations at the fusion node (we assume that the cognitive network knows its channel gains and has a better control over its system. This is a commonly made assumption in CR), the DualCUSUM at the fusion node remains unchanged. Also, for optimal implementation, in slot 1, each node initially captures $L_s + L_d$ samples. From then onwards in each slot j each node captures only L_s samples and uses the last L_d samples from slot $j-1$ to calculate $R_r(j,l,m)$. It can be shown that $R_r(j-1,l,m)$ and $R_r(j,l,m)$ remain uncorrelated. This can be roughly explained as any sample in these L_d samples will be correlated with some sample in slot $j-1$ or slot j, but not both.

C. GLR-CUSUM when timing offset, frequency offset and primary power are unknown

Now we assume that $\sigma_{s,l}^2$ is unknown. Additionally timing and frequency offset could also be present. Thus we cannot use R_r and need to use R instead. It is easy to see that when frequency offset is present, post change,

$R_r \sim N(\sigma_{s,l}^2 \cos(2\pi\varphi), \sigma_w^4/2L_c)$ and $R_i \sim N(\sigma_{s,l}^2 \sin(2\pi\varphi), \sigma_w^4/2L_c)$. As we now have a composite post change hypothesis we use the Generalized Likelihood Ratio (GLR)-CUSUM algorithm [7]. Under these conditions, the GLR test is

$$\underset{m\in\{0,1,\ldots L_d-1\}}{R(j,l,m)} = \frac{1}{L_c}\sum_{i=1}^{L_c} X(jL_s+i+m,l)X^*(jL_s+i+m+L_d,l), \qquad (16)$$

$$W_{j,l,m} = \max_{1\le t\le j}\left((\sum_{p=t}^{j} R_r(p,l,m))^2 + (\sum_{p=t}^{j} R_i(p,l,m))^2 \right) \Big/ \left((j-t+1)\sigma_w^4/L_c\right)$$

The rest of the steps at each secondary node are same as in (15) and at the fusion node, the DualCUSUM remains unchanged.

D. GLR-CUSUM when all impairments are present

We assume all the above mentioned impairments (including IQ imbalance) could be present and σ_w^2 and σ_s^2 remain unknown to the secondary node. While we can extend the GLR test to cover this scenario as well, we have found via simulations that it is better to first compensate for the IQ-imbalance in each slot using (11) and (12). Then we estimate the noise power as $\hat{\sigma}_{w,j,l}^2 = \sum_{p=1}^{j}\sum_{k=1}^{L_s} |X((p-1)L_s+k,l)|^2 / jL_s$. This approximation is valid under low SNR assumptions. Now, since the IQ imbalance can be assumed to have been corrected and we have an estimate of noise power $\hat{\sigma}_{w,j,l}^2$, we can use the setup of Sec.IV.C for the other impairments (timing offset, frequency offset and received primary power). Thus each node does the same steps as in Sec.IV.C using the estimated noise power in slot j, $\hat{\sigma}_{w,j,l}^2$.

E. Performance comparison

We compare the performance of the above algorithms in Table 4. There are 5 nodes. The SNR at each node is −10dB. The change time T (in units of OFDM symbols) is geometric with parameter $\rho = 0.004$. Rest of the parameters are as in Section III-A.

For comparison purposes, we have also simulated a cooperative snapshot detector which captures $M = 50$ OFDM symbols of data and detects the signal in the presence of all impairments and compensating for the same using the steps in Sec. III.E. Each node sends a 1 or 0 according to whether H_1 or H_0 is chosen. The fusion center uses the AND rule to decide between H_1 or H_0 as the AND rule works the best in the present setup. For the snapshot detector, we assume that fusion node has no noise (so that the fusion node knows how many secondaries have sent 1). For different values of P_{FA}, Expected Detection Delay (EDD) in units of OFDM symbols is shown in Table 4.

P_{FA}	(IV.A)	(IV.B)	(IV.C)	(IV.D)	Snapshot
0.1	10.15	18.27	24.71	28.15	64.16
0.075	11.43	19.82	28.07	31.01	67.46
0.05	12.6	22.09	31.42	34.95	72.35

Table 4: Cooperative spectrum sensing algorithms

We see that as the amount of uncertainty increases the performance degrades. Also from the last two columns we see that the sequential setup provides significant performance gains over the snapshot detector.

V. CONCLUSIONS

We have considered the problem of spectrum sensing of OFDM signals using cyclic prefix. We have analyzed the effect of some common impairments like timing and frequency offset, IQ-imbalance and transmit/noise power uncertainty and presented techniques to modify the CP-detector to work under these impairments. We have also proposed cooperative CP detection based algorithms and overcome the effects of these impairments. Future work involves analyzing behavior of CP-detector under multipath frequency selective fading.

REFERENCES

[1] C. Cordeiro, K. Challapali, D. Birru and N. S. Shankar, "IEEE 802.22: The First Worldwide Wireless Standard based on cognitive radios", in *Proc IEEE DySPAN*, Nov 2005, pp 328-37.

[2] S. Chaudhari, J. Lunden and V. Koivunen, "Colloborative Autocorrelation-Based Spectrum Sensing of OFDM Signals in Cognitive Radio", *42nd Conf. on Inform. Sc. and Systems*, Mar 2008, pp. 191-196.

[3] A. Sahai, N. Hoven, S.M. Mishra and R. Tandra, "Fundamental tradeoffs in robust spectrum sensing for opportunistic frequency reuse" in *Proc First Int.l Work. on Tech. and Policy for Accessing Spectrum*, Aug 2006.

[4] Y. Zhuan, J. Grosspietsch and G. Memik, "Spectrum Sensing using Cyclostationary Spectrum Density for Cognitive Radios", *IEEE Workshop on Signal Processing Systems*, Oct 2007, pp 1-6.

[5] K. B. Letaief and W.Zhang, "Cooperative Spectrum Sensing" *in Cognitive Wireless Communication Networks*, E. Hossain and V. K. Bhargava, (Eds.), Springer, June 2007.

[6] V. V. Veeravalli and J. Unnikrishnan, "Cooperative spectrum sensing for primary detection in Cognitive Radio", *IEEE Jnl. on Selected Topics in Signal Processing*, Feb 2008, pp. 18-27.

[7] A. Jayaprakasam and V. Sharma, "Cooperative robust sequential detection algorithms for spectrum sensing in cognitive radio", *Intl. Conf. on Ultra Modern Telcommunications (ICUMT)*, 2009.

[8] L. Lai, Y. Fan and H.V. Poor, "Quickest Detection in Cognitive Radio: A sequential change detection framework", *IEEE GlobeCOMM*, 2008.

[9] H. Li, C. Li and H. Dai, "Quickest spectrum sensing in cognitive radio", in *Proc IEEE CISS, Mar 2008*, pp 203-208.

[10] V. Sharma and A. Jayaprakasam, "An Efficient algorithm for cooperative Spectrum Sensing in Cognitive Radio Networks", in *Proc National Communications Conference (NCC)*, Jan 2009.

[11] S. Chaudhari, V. Koivunen and H.V.Poor, "Autocorrelation based decentralized sequential detection of OFDM signals in cognitive radios", *IEEE Trans. on Signal Processing*, Vol. 57, 2009, pp. 2690-2700.

[12] S-Y Tu, K-C chen and Ramjee Prasad, "Spectrum Sensing of OFDMA systems for cognitive radio networks", *IEEE Trans. Vehicular technolgy*, Vol. 58, Issue 7, Sep 2009, pp. 3410-3425.

[13] R. Wang, V.K.N. Lace, L. Lv and B. Chen, "Joint cross-layer scheduling and spectrum sensing for OFDMA cognitive radio systems", IEEE Trans. Wireless Commn., Vol 8, 2009, pp. 2410-2416.

[14] J. V. Beek, M. Sandell and P. O. Borjesson, "ML Estimation of Time and Frequency Offset in OFDM Systems", *IEEE trans. on Signal Processing*, Vol. 45, No. 7, July 1997.

[15] I. Held, O. Klein, A. Chen and V. Ma, "Low Complexity Digital IQ Imbalance Correction in OFDM WLAN Receivers", *IEEE Vehicular Technolgy Conf.*, May 2004, Vol. 2, pp. 1172-1176.

[16] T. Banerjee, V. Kavitha and V. Sharma, "Energy efficient change detection over a MAC using physical layer fusion", in *Proc of IEEE ICASSP*, Mar 2008, pp 2501-2504.

[17] E. S. Page, "Continuous Inspection Schemes", *Biometrika*, Vol 41, No 1/2 , pp 100-115, June 1954.