Transmission of Correlated Sources Over a Fading Multiple Access Channel

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Abstract— **In this paper we address the problem of transmission of correlated sources over a fading multiple access channel (MAC). We provide sufficient conditions for transmission with given distortions. Next these conditions are specialized to a Gaussian MAC (GMAC). Transmission schemes for discrete and Gaussian sources over a fading GMAC are considered. Various power allocation strategies are also compared.**
Keywords: Fading MAC, Power allocation, Random TDMA, Amplify and Forward, Correlated sources.

I. Introduction and survey

Sensor nodes are often deployed for monitoring a random field. Due to the spatial proximity of the nodes, sensor observations are correlated. One often needs to transmit these observations to a fusion center through wireless links which experience multipath fading. A fundamental building block for such a network is a fading Multiple Access Channel (MAC). We study this system in this paper.

In the following we survey the related literature. Cover, El Gamal and Salehi [3] provided sufficient conditions for transmitting losslessly discrete correlated observations over a discrete MAC. They also show that unlike for independent sources, the source-channel separation does not hold. These techniques were extended to more general models with discrete sources and channels and lossless transmission in [1]. References [15], [20] extend the result in [3] and obtain sufficient conditions for lossy transmission of correlated sources over a MAC with side information. Joint source-channel coding schemes for transmission of correlated sources over a MAC are also discussed in [5] and [10].

The capacity of a fading Gaussian channel with channel state information (CSI) at the transmitter and receiver and at the receiver alone are provided in [6]. It was shown that optimal power adaptation when CSI is available both at the transmitter and the receiver is 'water filling' in time.

The capacity region of the GMAC with independent inputs is available in [4]. The distributed Gaussian source coding problem is discussed in [12], [21]. Exact rate region for two users is provided in [21]. Hanly and Tse ([18], [19]) have generalized the results on a GMAC with independent inputs to the case with flat fading and CSI available at both the transmitter and receiver. They obtain the Shannon (ergodic) capacity and delay limited capacity.

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The multi-access fading channels with independent inputs were also considered in the excellent survey [2]. They also show that unlike in the single user case in the multi-access realm the optimal power control yields a substantial gain in capacity. The optimal power allocation strategy for the symmetric case is to allow the user with the best channel to transmit at a time (Random TDMA) ([8]). The instantaneous power allocated to a user is by the well known 'water-filling' algorithm in time.

The optimal strategy is also valid when there is different average power constraints on different users. The only change being that the fading values are normalized by the Lagrange's coefficients [8]. The extension of this strategy to frequency selective channels is given in [9].

Multiple-access techniques for fading cellular uplink model with adjacent cell interference are discussed in [17].

An explicit characterization of the ergodic capacity region and a simple encoding-decoding scheme for a fading GMAC with common data is given in [11] for lossless transmission. Optimum power allocation schemes are also provided.

This paper makes the following contributions. Sufficient conditions for lossless and lossy transmission of correlated sources over a fading MAC are obtained. The source alphabet and/or the channel alphabet can be discrete or continuous. The conditions are specialized for GMAC and an optimal power allocation policy is derived. It is shown that the 'random TDMA' (i.e., optimum power allocation for a fading MAC with independent inputs) is strictly sub-optimal in this case. Suitable examples are given and the optimum power allocation policy is compared with other commonly used policies.

The paper is organized as follows. Sufficient conditions for transmission of correlated sources over a fading MAC are provided in Section II. These conditions are specialized to a fading GMAC in Section III. Section IV obtains the optimal power allocation policy and its comparison with other power allocation policies. Section V concludes the paper. Proof of the main result is given in the Appendix.

II. Transmission of correlated sources over a fading MAC

In this section we consider the transmission of memoryless dependent sources, through a memoryless fading multiple access channel. The sources and/or the channel input/output alphabets can be discrete or continuous. The transmitters and the receiver have perfect knowledge of the fade state of the channel at that time.

We consider two sources (U_1, U_2) with a known joint distribution $F(u_1, u_2)$. The random vector sequence $\{(U_{1n}, U_{2n}),\ n \geq 1\}$ formed from the source outputs with distribution F is independent identically distributed (*iid*) in time. The sources transmit their code words X_i's to a single decoder through a memoryless, flat, fast fading multiple access channel. Let (H_{1n}, H_{2n}) be the fade state at time $n,\ n \geq 1$. We assume $\{(H_{1n}, H_{2n}),\ n \geq 1\}$ to be iid, although (H_{1n}, H_{2n}) can be dependent and can be discrete or continuous valued. The channel output Y has distribution $p(y|x_1, x_2, h_1, h_2)$ if x_1 and x_2 are transmitted at that time and the channel is in the fade state (h_1, h_2). The decoder receives Y and estimates the sensor observations U_i as $\hat{U}_i,\ i = 1, 2$.

It is of interest to find encoders and a decoder such that $\{U_{1n}, U_{2n}, n \geq 1\}$ can be transmitted over the given fading MAC with $E[d_i(U_i, \hat{U}_i)] \leq D_i,\ i = 1, 2$ where d_i are non-negative distortion measures and D_i are the given distortion constraints. We will assume that d_i are such that $d_i(u, u') = 0$ if and only if $u = u'$. If the distortion measures are unbounded we also assume that there exist $u_i^*,\ i = 1, 2$ such that $E[d_i(U_i, u_i^*)] < \infty,\ i = 1, 2$.

Source channel separation does not hold in this case.

We will denote $U_{ij}, j = 1, 2...n$ by $U_i^n, i = 1, 2$.

$Definition$: The sources (U_1^n, U_2^n) can be transmitted over the fading multiple access channel with distortions $\mathbf{D} \triangleq (D_1, D_2)$ if for any $\epsilon > 0$ there is an n_0 such that for all $n > n_0$ there exist encoders $f_{E,i}^n : \mathcal{U}_i^n \times \mathcal{H}_1^n \times \mathcal{H}_2^n \rightarrow \mathcal{X}_i^n,\ i = 1, 2$ and a decoder $f_D^n : \mathcal{Y}^n \times \mathcal{H}_1^n \times \mathcal{H}_1^n \rightarrow (\hat{\mathcal{U}}_1^n, \hat{\mathcal{U}}_2^n)$ such that $\frac{1}{n} E\left[\sum_{j=1}^n d(U_{ij}, \hat{U}_{ij})\right] \leq D_i + \epsilon,\ i = 1, 2$ where $(\hat{U}_1^n, \hat{U}_2^n) = f_D(Y^n, H_1^n, H_2^n)$ and $\mathcal{U}_i,\ \mathcal{H}_i,\ \mathcal{X}_i,\ \mathcal{Y},\ \hat{\mathcal{U}}_i$ are the sets in which $U_i,\ H_i,\ X_i,\ Y$ and $\hat{U}_i$ take values.

Since the MAC is memoryless, $p(y^n|x_1^n, x_2^n, h_1^n, h_2^n) = \prod_{j=1}^n p(y_j|x_{1j}, x_{2j}, h_{1j}, h_{2j})$. In the following $X \leftrightarrow Y \leftrightarrow Z$ will indicate that (X, Y, Z) forms a Markov chain.

Now we state the main Theorem.

Theorem 1: A source can be transmitted over a fading multiple access channel with distortions (D_1, D_2) if there exist random variables (W_1, W_2, X_1, X_2) such that

1. $$p(u_1, u_2, w_1, w_2, x_1, x_2, y, h_1, h_2) = p(u_1, u_2)p(w_1|u_1)p(w_2|u_2) p(x_1|w_1, h_1, h_2)p(x_2|w_2, h_1, h_2)p(y|x_1, x_2, h_1, h_2).$$

2. There exists a function $f_D : \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{H}_1 \times \mathcal{H}_2 \rightarrow (\hat{\mathcal{U}}_1 \times \hat{\mathcal{U}}_2)$ such that $E[d(U_i, \hat{U}_i)] \leq D_i,\ i = 1, 2$ and the constraints

$$\begin{aligned} I(U_1; W_1|W_2) &< I(X_1; Y|X_2, W_2, H_1, H_2), \\ I(U_2; W_2|W_1) &< I(X_2; Y|X_1, W_1, H_1, H_2), \quad (1) \\ I(U_1, U_2; W_1, W_2) &< I(X_1, X_2; Y|H_1, H_2), \end{aligned}$$

are satisfied where $\mathcal{W}_i$ is the set in which W_i take values. ■

If the channel input alphabets are continuous valued then the $X_i s$ should also satisfy given power constraints $E[X_i^2] \leq \overline{P}_i,\ i = 1, 2$.

In Theorem 1 it is possible to include other distortion constraints. For example, in addition to the bounds on $E[d(U_i, \hat{U}_i)]$ one may want a bound on the joint distortion $E[d((U_1, U_2), (\hat{U}_1, \hat{U}_2))]$. Then the only modification needed in the statement of the above theorem is to include this also as a condition in defining f_D.

The proof of the theorem is given in the Appendix.

The proof extends directly to the multi-user case (with the number of users > 2). Let $\mathcal{S} = 1, 2, ..., M$ be the set of sources with joint distribution $p(u_1, u_2...u_M)$. $H_{\mathcal{S}}$ denotes the set $\{H_1, H_2, ..., H_M\}$.

Theorem 2: Sources $(U_i^n, i \in \mathcal{S})$ can be communicated in a distributed fashion over the memoryless fading multiple access channel $p(y|x_i, i \in \mathcal{S})$ with distortions $(D_i, i \in \mathcal{S})$ if there exist auxiliary random variables $(W_i, X_i, i \in \mathcal{S})$ satisfying

1. $$p(u_i, w_i, x_i, y, h_i, i \in \mathcal{S}) = p(u_i, i \in \mathcal{S})p(y|x_i, h_i, i \in \mathcal{S}) \prod_{j \in \mathcal{S}} p(w_j|u_j)p(x_j|w_j, h_{\mathcal{S}})$$

2. There exists a function $f_D : \prod_{j \in \mathcal{S}} \mathcal{W}_j \times \mathcal{H} \rightarrow (\hat{\mathcal{U}}_i, i \in \mathcal{S})$ such that $E[d(U_i, \hat{U}_i)] \leq D_i,\ i \in \mathcal{S}$ and the constraints

$$I(U_A; W_A|W_{A^c}) < I(X_A; Y|X_{A^c}, W_{A^c}, H_{\mathcal{S}}) \quad (2)$$

are satisfied (in case of continuous channel alphabets we also need the power constraints $E[X_i^2] \leq P_i,\ i = 1, ..., M$) for all $A \subset \mathcal{S}$.

■

The proof is along the lines of the proof of Theorem 1 and is omitted for the sake of brevity.

One of the main problems in using (1) in practice is in obtaining efficient, explicit W_1, W_2, X_1, X_2, f_D for a given $(U_1, U_2), (D_1, D_2)$ and the channel. In the rest of this paper we obtain good coding-decoding schemes for a GMAC.

III. Fading Gaussian MAC

In a fading Gaussian MAC the channel output Y_n at time n is given by $Y_n = H_{1n}X_{1n} + H_{2n}X_{2n} + N_n$ where X_{1n} and X_{2n} are the channel inputs at time n and $\{N_n\}$ is iid with a Gaussian distribution and is independent of X_{1n} and X_{2n}. Also, $E[N_n] = 0$ and $var(N_n) = \sigma_N^2$. H_{1n} and H_{2n} are the fade states of the channel at time n. The power constraints on the channel inputs are $E[X_i^2] \leq \overline{P}_i,\ i = 1, 2$. The distortion measure will be Mean Square Error (MSE). Let $\tilde{\rho}$ be the correlation between the channel inputs X_1, X_2.

From the maximum correlation theorem ([16]), $\tilde{\rho} \leq \rho$, where ρ is the correlation between U_1 and U_2. Also, it is shown in [14] that if (U_1, U_2) are the correlated sources and $X_1 \leftrightarrow U_1 \leftrightarrow U_2 \leftrightarrow X_2$ where X_1 and X_2 are jointly Gaussian, then the correlation between (X_1, X_2) satisfies

$$
\begin{aligned}
I(U_1;W_1|W_2) &< 0.5E\left[\log\left(1+\frac{|H_1|^2P_1(H_1,H_2)(1-\tilde{\rho}^2)}{{\sigma_N}^2}\right)\right],\\
I(U_2;W_2|W_1) &< 0.5E\left[\log\left(1+\frac{|H_2|^2P_2(H_1,H_2)(1-\tilde{\rho}^2)}{{\sigma_N}^2}\right)\right], \qquad (3)\\
I(U_1,U_2;W_1,W_2) &< 0.5E\left[\log\left(1+\frac{|H_1|^2P_1(H_1,H_2)+|H_2|^2P_2(H_1,H_2)+2|H_1||H_2|\tilde{\rho}\sqrt{P_1(H_1,H_2)P_2(H_1,H_2)}}{{\sigma_N}^2}\right)\right].
\end{aligned}
$$

$\rho^2 \leq 1 - 2^{-2I(U_1,U_2)}$. The proof is using data processing inequality and the result sometimes gives a tighter upper bound than the maximum correlation theorem. For example, consider (U_1, U_2) with the joint distribution: $P(U_1 = 0; U_2 = 0) = P(U_1 = 1; U_2 = 1) = 1/3; P(U_1 = 1; U_2 = 0) = P(U_1 = 0; U_2 = 1) = 1/6$. The correlation between the sources is 0.33 but from the above result, the correlation between (X_1, X_2) cannot exceed 0.327.

For this GMAC, following the experience in [14] we relax the first two inequalities in (1) to make them more explicit. These are then used to obtain efficient signaling schemes to satisfy (1). For this the R.H.S. of the first two inequalities in (1) are replaced by upper bounds $I(X_1; Y|X_2, H_1, H_2)$ and $I(X_2; Y|X_1, H_1, H_2)$ respectively. It is shown in [14] that these upper bounds are quite tight whenever these two inequalities are active (generally it is the third inequality which is tight). Also, it is shown in [14] that for a given (h_1, h_2), these upper bounds and the R.H.S. of the third inequality in (1) are maximized by choosing (X_1, X_2) to be zero mean, jointly Gaussian r.v.s with $E[X_i^2] = P_i(h_1, h_2)$. If such (X_1, X_2) have correlation $\tilde{\rho}$ these three bounds provide (3). We also need to choose the power control policies $P_i(h_1, h_2)$ such that the average power constraints

$$E[P_i(H_1, H_2)] \leq \overline{P}_i,\ i = 1, 2, \qquad (4)$$

are satisfied. This motivates us to consider Gaussian coding schemes.

An advantage of (3) is that we will be able to obtain explicit source-channel coding schemes to satisfy (3). These may be difficult to identify from (1) itself. Once we have obtained these coding schemes we can verify the sufficient conditions (1) themselves. If satisfied these will ensure that the coding schemes can ensure transmission with given distortions. If not, one can change $\tilde{\rho}$ to finally satisfy (1). Thus in the rest of the paper we consider some power allocation policies along with Gaussian signaling schemes which can be used to satisfy the conditions (1).

An important special case is when the correlated sources (U_1, U_2) are $U_1 = (Z_1, Z_0)$, $U_2 = (Z_2, Z_0)$ where Z_0, Z_1, Z_2 are independent. Z_0 represents the common information. We vector quantize Z_1^n, Z_2^n, Z_0^n separately into W_1^n, W_2^n and W_0^n. These in turn are independently coded into Gaussian codebooks. The corresponding codewords are scaled into X_1^n, X_2^n, X_0^n with powers $(1-\alpha)\overline{P}_1, (1-\beta)\overline{P}_2$ reserved for X_0^n at the two encoders and the rest on X_1^n and X_2^n respectively. Then using conditions (1) for three users with lossless transmission we can recover the capacity result in [11].

IV. Optimal power allocation

We consider an optimal power allocation policy such that the R.H.S. in the third inequality of (3) is maximized and the other conditions are satisfied. This is done because often the third inequality is the constraining condition. The optimal power allocation policy is obtained numerically. It depends upon $\tilde{\rho}$ which inturn depends on the source correlation ρ.

To find a $\tilde{\rho}$ such that all the three inequalities are satisfied by the optimal policy, we can use the following procedure. We consider an iterative algorithm in which the channel correlation $(\tilde{\rho})$ is chosen in such a way that the third inequality is satisfied. Then we check for the other two inequalities. If they fail then the $\tilde{\rho}$ is decreased so that all the three conditions are satisfied, if possible.

The optimal policy is compared with Random TDMA (RTDMA), Modified Random TDMA (MRTDMA) and Uniform Power Allocation (UPA) policies defined below. In RTDMA only the user having the best channel conditions transmits (as in [2], [8], [9]). If the best channel state is obtained by more than one user, then one of these users is selected with equal probability. Once a user is chosen the power allocation is by water-filling over the channel states of that user. This policy maximizes the sum of channel rates for independent sources and symmetric channel statistics and power constraints. Thus for $\tilde{\rho} = 0$ it equals the optimal policy under symmetric conditions because then the third inequality gives the sum rates. In MRTDMA also only the user having the best channel conditions transmits. However if the fading values are same for both the users then the total power is split and both the users transmit simultaneously. In UPA both the users transmit all the time at powers P_1 and P_2.

The optimal power allocation policy expends equal power on both the users when the fade states are same. When the users have unequal fading, then the user with worse channel gets lesser fraction of the power. Its share increases with $\tilde{\rho}$ and become close to 0.5 when the two fade states come closer.

In the following we compare these power allocation policies on discrete and Gaussian sources. In particular we show that RTDMA may not be optimal for correlated sources even for symmetric channel and power constraints. Also, that even though MRTDMA may improve over RTDMA significantly, it is still suboptimal.

A. Discrete Sources

We show the sub-optimality of RTDMA and MRTDMA via an example.

Consider the lossless transmission of discrete sources (U_1, U_2) over a fading GMAC. Such a system is most commonly encountered in practice. The sources (U_1, U_2) have joint distribution given by $P(U_1 = 0; U_2 = 0) = P(U_1 = 1; U_2 = 1) = P(U_1 = 0; U_2 = 1) = 1/3; P(U_1 = 1; U_2 = 0) = 0$. The fade states h_1, h_2 take values in $(1, 0.5)$ with equal probability and are independent of each other. The power constraints on the channel inputs (X_1, X_2) are $(5, 5)$. The channel noise is zero mean with unit variance.

For lossless transmission the L.H.S. in (3) become $H(U_1|U_2)$, $H(U_2|U_1)$ and $H(U_1, U_2)$ respectively and they evaluate to 0.667, 0.667 and 1.585. Let the sources be mapped to channel codewords with correlation $\tilde{\rho} = 0.3$. Such correlation preserving mappings are discussed in [14]. If we use UPA the R.H.S. in the third inequality evaluates to 1.5030. If we consider RTDMA the R.H.S. evaluates to 1.5273. Thus in both the cases the third inequality is violated, and lossless transmission with these power control schemes may not be possible. With MRTDMA the R.H.S. in the third inequality improves to 1.6036 and the optimal scheme provides 1.6071. The R.H.S in the first two inequalities evaluate to 0.8627 for the MRTDMA and to 0.8755 with the optimal policy. From the coding scheme in [14] discussed above the original bounds in the first two inequalities of (1) evaluate to 0.873 for optimal power allocation and 0.8315 for MRTDMA. Hence the first two inequalities are also satisfied. This ensures that the sources can be transmitted losslessly with these two power allocation schemes.

Hence from the above example we see that unlike for independent sources the RTDMA does not perform optimally with correlated sources even for a symmetric system.

B. Gaussian Sources

Consider the transmission of correlated Gaussian sources over a GMAC. The sources are assumed to have zero mean, unit variance and correlation ρ. The power constraints on the channel inputs are $(\overline{P_1}, \overline{P_2})$ and channel noise variance is unity. The performance measure is to find the minimum distortion at the decoder for given sources, power constraints and channel noise variance. The distortion criterion is mean square error (MSE).

We consider three joint source-channel coding schemes discussed in [13]. The schemes discussed below have been used in [5], [7] and [10] also. Each of these coding-decoding scheme is used with power allocation schemes RTDMA, MRTDMA, UPA and the optimal scheme obtained above from (3) which minimizes the distortions.

The first scheme is amplify and forward (AF) where the sources are amplified to the powers allocated for each state and sent. The power allocated to each state depends on the power allocation policy used. The channel inputs preserve the correlation of the source outputs (U_1, U_2). The decoding is performed by estimation of the sources (U_1, U_2) from the channel output Y as $(\hat{U}_1, \hat{U}_2) = E[(U_1, U_2)|Y]$. This scheme was shown to be optimal in [10] for symmetric case at low SNR on channels without fading.

In the second scheme called Separation Based (SB), the source and channel coding are separated. The source coding is done by vector-quantization followed by Slepian-Wolf compression ([4]) and the compressed outputs are mapped to independent channel r.v.s. These are amplified to the state dependent powers to obtain (X_1, X_2) and sent.

In Lapidoth-Tinguely (LT) scheme (developed in [10]) the sources are vector quantized and mapped to correlated Gaussian codewords.

The decoding for both the above schemes is jointly typical decoding followed by estimation of the sources.

We compare the power allocation strategies optimal, RTDMA, MRTDMA and UPA for these joint source-channel coding schemes for various values of source correlations. We will take $(\overline{P_1}, \overline{P_2}) = (5, 5)$. The fading processes are as in Section IV-A. The mean distortion of each user is provided in Figs. 1-3. Because of symmetry the mean distortion will be same for each user. Fig. 1 provides the results for AF, Fig. 2 for SB and Fig. 3 for LT. For comparison we have also included the results for the channel with no fading.

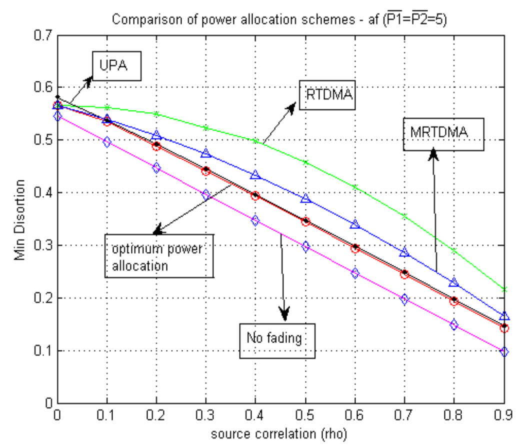

Fig. 1. Minimum distortion for AF, $\overline{P_1} = \overline{P_2} = 5$

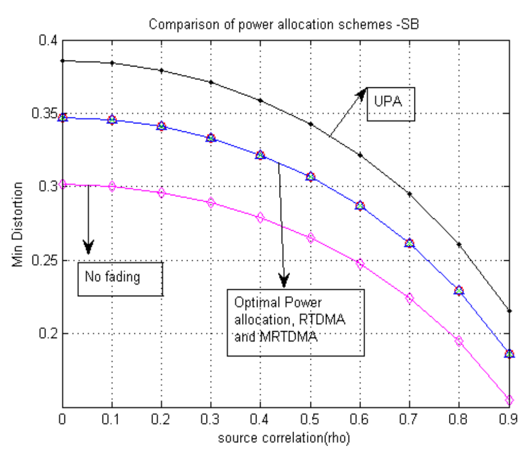

Fig. 2. Minimum distortion for SB, $\overline{P_1} = \overline{P_2} = 5$

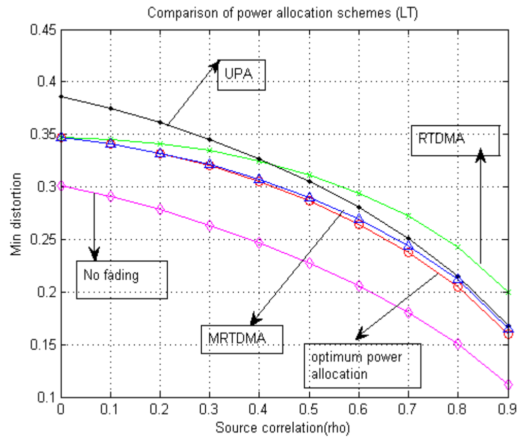

Fig. 3. Minimum distortion for LT, $\overline{P_1} = \overline{P_2} = 5$

From Figs. 1-3 we find that RTDMA performs worse than the optimal scheme and MRTDMA for AF and LT. It is to be also noted that in AF, RTDMA is sub-optimal even when the sources are independent. This is because water-filling does not give the optimal power allocations that minimizes the distortion in AF. In SB the optimal power allocation scheme is the RTDMA itself as the sources after Slepian-Wolf compression become (asymptotically) independent. In AF, UPA is close to the optimal. In LT, MRTDMA performs close to the optimal.

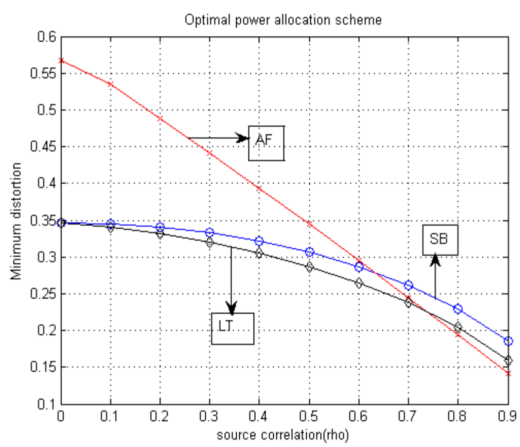

Fig. 4. Comparison of the optimum power allocation schemes, $\overline{P_1} = \overline{P_2} = 5$

In Fig. 4 we have plotted the performance of the optimal scheme for AF, SB and LT. It can be seen that LT performs better than SB for all ρ and AF performs better than the other two schemes at high ρ. This is due to the fact that, for the symmetric case ($\overline{P_1} = \overline{P_2} = \overline{P}$), for $\frac{\overline{P}}{\sigma_N^2} \leq \frac{\rho}{1-\rho^2}$ AF is optimal([10]). These are in conformance with the conclusions in [13].

One of the performance measures is to maximize the R.H.S. in the third inequality of (3). The optimal power allocation policy discussed in the beginning of this section maximizes this. We compare RTDMA, MRTDMA and UPA for this criterion.

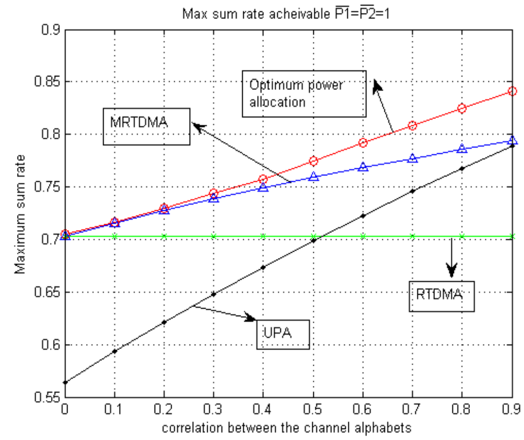

Fig. 5. Maximum sum rate for different power allocation policies with $\overline{P_1} = \overline{P_2} = 1$

We take $\overline{P_1} = \overline{P_2} = 1$ with the fading as above. The channel correlation ($\tilde{\rho}$) achievable depends on the source correlations and the scheme used. We compare the power allocation policies for different $\tilde{\rho}$ in Fig. 5.

From the figure it is clear that unlike in the case of independent inputs, the RTDMA based power allocation is strictly suboptimal for correlated sources. We also see that a slight modification in RTDMA resulting in MRTDMA can give performance close to the optimal scheme. At high channel correlation even the UPA performs better than the RTDMA.

V. Conclusions

In this paper, sufficient conditions for transmission of correlated sources over a fading MAC are provided. These conditions are specialized to a GMAC and an optimal power allocation policy is discussed. The optimal policy is compared with other well known policies in literature and it is found that the 'random TDMA', optimal for independent sources, does not perform optimally for transmission of correlated sources over a GMAC.

Appendix
Proof of Theorem 1

First we consider discrete sources and channel inputs. Comments to include the continuous sources/channel inputs are provided at the end of the proof.

The scheme involves distributed quantization (W_1^n, W_2^n) of the sources followed by a correlation preserving mapping to the channel codewords depending on the channel state. The decoding approach involves first decoding the quantized version (W_1^n, W_2^n) of (U_1^n, U_2^n), and then obtaining estimate $(\hat{U}_1, \hat{U}_2)$ as a function of (W_1^n, W_2^n).

We show the achievability of all points in the rate region (1).

Proof: Fix $p(w_1|u_1)$, $p(w_2|u_2)$, $p(x_1|w_1, h_1, h_2)$, $p(x_2|w_2, h_1, h_2)$ as well as f_D^n satisfying the distortion constraints.

Codebook generation: Let $R_i^{'} = I(U_i;W_i) + \delta,\ i = 1,2$ for some $\delta > 0$. Generate $2^{nR_i^{'}}$ codewords of length n, sampled iid from the marginal distribution $p(w_i),\ i = 1,2$. For each w_i^n and (h_1^n, h_2^n) independently generate sequence X_i^n according to $\prod_{j=1}^{n} p(x_{ij}|w_{ij}, h_{1j}, h_{2j}),\ i = 1,2$. Call these sequences $x_i^n(w_i^n, h_1^n, h_2^n), i \in 1,2$. Reveal the codebooks to the encoders and the decoder.

Encoding: For $i \in \{1,2\}$, given the source sequence U_i^n and h_1^n, h_2^n, the i^{th} encoder looks for a codeword W_i^n such that $(U_i^n, W_i^n) \in T_\epsilon^n(U_i, W_i)$ and then transmits $X_i^n(W_i^n, h_1^n, h_2^n)$ where $T_\epsilon^n(.)$ is the set of ϵ-typical sequences ([4]) of length n.

Decoding: Upon receiving Y^n, for a given (h_1^n, h_2^n) the decoder finds the unique (W_1^n, W_2^n) pair such that $(W_1^n, W_2^n, x_1^n(W_1^n, h_1^n, h_2^n), x_2^n(W_2^n, h_1^n, h_2^n), Y^n) \in T_\epsilon^n$. If it fails to find such a unique pair, the decoder declares (u_1^{*n}, u_2^{*n}).

In the following we show that the probability of error for the above encoding, decoding scheme tends to zero as $n \to \infty$. By Markov Lemma ([4]), $P\{(U_1^n, U_2^n, W_1^n(U_1^n), W_2^n(U_2^n), X_1^n(W_1^n, h_1^n, h_2^n), X_2^n(W_2^n, h_1^n, h_2^n), Y^n) \in T_\epsilon^n\} \to 1$ as $n \to \infty$. The error can occur because of the following three events **E1**-**E3**. We show that $P(\mathbf{Ei}) \to 0$, for $i = 1,2,3$. The state sequence (h_1^n, h_2^n) is known both at the encoder and the decoder. For simplicity we take $\delta = \epsilon$.

E1 The encoders do not find the codewords. However from rate distortion theory [4], P. 356, $\lim_{n\to\infty} P(E_1) = 0$ if $R_i^{'} > I(U_i;W_i), i \in 1,2$.

E2 There exists another codeword $\hat{w}_1^n$ such that $(\hat{w}_1^n, W_2^n, x_1^n(\hat{w}_1^n, h_1^n, h_2^n), x_2^n(W_2^n, h_1^n, h_2^n), Y^n) \in T_\epsilon^n$. Define $\alpha \triangleq (\hat{w}_1^n, W_2^n, x_1^n(\hat{w}_1^n, h_1^n, h_2^n), x_2^n(W_2^n, h_1^n, h_2^n), Y^n)$. Then,

$$\begin{aligned} P(\mathbf{E2}) &= P\{\text{There is } \hat{w}_1^n \neq w_1^n : \alpha \in T_\epsilon^n\} \\ &\leq \sum_{\hat{w}_1^n \neq W_1^n : (\hat{w}_1^n, W_2^n) \in T_\epsilon^n} P\{\alpha \in T_\epsilon^n\} \end{aligned} \tag{5}$$

Denote $\{(x_1^n(.), x_2^n(.), y^n) : \alpha \in T_\epsilon^n\}$ by A. The probability term inside the summation in (5) is

$$\begin{aligned} &\leq \sum_A P\{x_1^n(\hat{w}_1^n, h_1^n, h_2^n), x_2^n(w_2^n, h_1^n, h_2^n), y^n | \hat{w}_1^n, w_2^n\} \\ &\leq \sum_A P\{x_1^n(\hat{w}_1^n, h_1^n, h_2^n) | \hat{w}_1^n, h_1^n, h_2^n\} \\ &\qquad P\{x_2^n(w_2^n, h_1^n, h_2^n), y^n | w_2^n, h_1^n, h_2^n\} \\ &\leq \sum_A 2^{-n\{H(X_1|W_1,H_1,H_2) + H(X_2,Y|W_2,H_1,H_2) - 4\epsilon\}} \\ &\leq 2^{nH(X_1,X_2,Y|W_1,W_2,H_1,H_2)} \\ &\quad 2^{-n\{H(X_1|W_1,H_1,H_2) + H(X_2,Y|W_2,H_1,H_2) - 6\epsilon\}} \end{aligned}$$

But from hypothesis, we have

$$\begin{aligned} &H(X_1, X_2, Y|W_1, W_2, H_1, H_2) - H(X_1|W_1, H_1, H_2) \\ &\qquad -H(X_2, Y|W_2, H_1, H_2) \\ &= -I(X_1; Y|X_2, W_2, H_1, H_2). \end{aligned}$$

Hence,

$$\begin{aligned} &P\{(\hat{w}_1^n, W_2^n, x_1^n(\hat{w}_1^n, h_1^n, h_2^n), x_2^n(W_2^n, h_1^n, h_2^n), Y^n) \in T_\epsilon^n\} \\ &\qquad \leq 2^{-n\{I(X_1;Y|X_2,W_2,H_1,H_2) - 6\epsilon\}}. \end{aligned} \tag{6}$$

Then from (5)

$$\begin{aligned} P(\mathbf{E2}) &\leq \sum_{\hat{w}_1^n \neq w_1^n : (\hat{w}_1^n, w_2^n) \in T_\epsilon^n} 2^{-n\{I(X_1;Y|X_2,W_2,H_1,H_2) - 6\epsilon\}} \\ &\leq |\{\hat{w}_1^n : (\hat{w}_1^n, w_2^n) \in T_\epsilon^n\}| 2^{-n\{I(X_1;Y|X_2,W_2,H_1,H_2) - 6\epsilon\}} \\ &\leq |\{\hat{w}_1^n\}| P\{\hat{w}_1^n, w_2^n) \in T_\epsilon^n\} 2^{-n\{I(X_1;Y|X_2,W_2,H_1,H_2) - 6\epsilon\}} \\ &\leq 2^{n\{I(U_1;W_1) + \epsilon\}} 2^{-n\{I(W_1;W_2) - \epsilon\}} \\ &\qquad 2^{-n\{I(X_1;Y|X_2,W_2,H_1,H_2) - 6\epsilon\}} \\ &= 2^{n\{I(U_1;W_1|W_2)\}} 2^{-n\{I(X_1;Y|X_2,W_2,H_1,H_2) - 8\epsilon\}}. \end{aligned} \tag{7}$$

The R.H.S of the above inequality tends to zero if $I(U_1;W_1|W_2) < I(X_1;Y|X_2,W_2,H_1,H_2)$. In (7) we have used the fact that

$$I(U_1;W_1) - I(W_1;W_2) = I(U_1;W_1|W_2).$$

Similarly, by symmetry of the problem we require

$$I(U_2;W_2|W_1) < I(X_2;Y|X_1,W_1,H_1,H_2). \tag{8}$$

E3 There exist other codewords $\hat{w}_1^n$ and $\hat{w}_2^n$ such that $\alpha \triangleq (\hat{w}_1^n, \hat{w}_2^n, x_1^n(\hat{w}_1^n, h_1^n, h_2^n), x_2^n(\hat{w}_2^n, h_1^n, h_2^n), y^n) \in T_\epsilon^n$. Then,

$$\begin{aligned} P(\mathbf{E3}) &= P\{\text{There is } (\hat{w}_1^n, \hat{w}_2^n) \neq (w_1^n, w_2^n) : \alpha \in T_\epsilon^n\} \\ &\leq \sum_{(\hat{w}_1^n, \hat{w}_2^n) \neq (w_1^n, w_1^n) : (\hat{w}_1^n, \hat{w}_2^n) \in T_\epsilon^n} P\{\alpha \in T_\epsilon^n\}. \end{aligned} \tag{9}$$

Denote $(x_1^n(.), x_2^n(.), y^n) : \alpha \in T_\epsilon^n$ by A. The probability term inside the summation in (9) is

$$\begin{aligned} &\leq \sum_A Pr\{x_1^n(\hat{w}_1^n, h_1^n, h_2^n), x_2(w_2^n, h_1^n, h_2^n), y^n | \hat{w}_1^n, \hat{w}_2^n\} \\ &\leq \sum_A Pr\{x_1^n(\hat{w}_1^n, h_1^n, h_2^n) | \hat{w}_1^n, h_1, h_2\} \\ &\qquad Pr\{x_2^n(\hat{w}_2^n, h_1^n, h_2^n) | \hat{w}_2^n, h_1^n, h_2^n\} Pr\{y^n | h_1^n, h_2^n\} \\ &\leq \sum_A 2^{-n\{H(X_1|W_1,H_1,H_2) + H(X_2|W_2,H_1,H_2) + H(Y|H_1,H_2) - 5\epsilon\}} \\ &\leq 2^{nH(X_1,X_2,Y|W_1,W_2,H_1,H_2)} \\ &\quad 2^{-n\{H(X_1|W_1,H_1,H_2) + H(X_2|W_2,H_1,H_2) + H(Y|H_1,H_2) - 7\epsilon\}}. \end{aligned}$$

But from hypothesis, we have

$$\begin{aligned} &H(X_1, X_2, Y|W_1, W_2, H_1, H_2) - H(X_1|W_1, H_1, H_2) \\ &\qquad -H(X_2|W_2, H_1, H_2) - H(Y|H_1, H_2) \\ &= -I(X_1, X_2; Y|H_1, H_2) \end{aligned}$$

Hence,

$$\begin{aligned} &Pr\{(\hat{w}_1^n, \hat{w}_2^n, x_1^n(\hat{w}_1^n, h_1^n, h_2^n), x_2^n(\hat{w}_2^n, h_1^n, h_2^n), y^n) \in T_\epsilon^n\} \\ &\qquad \leq 2^{-n\{I(X_1,X_2;Y|H_1,H_2) - 7\epsilon\}}. \end{aligned} \tag{10}$$

Then from (9)

$$P(\mathbf{E3}) \leq \sum_{\substack{(\hat{w}_1^n,\hat{w}_2^n)\neq(w_1^n,w_1^n):\\(\hat{w}_1^n,\hat{w}_2^n)\in T_\epsilon^n}} 2^{-n\{I(X_1,X_2;Y|H_1,H_2)-7\epsilon\}}$$
$$\leq |\{(\hat{w}_1^n,\hat{w}_2^n):(\hat{w}_1^n,\hat{w}_2^n)\in T_\epsilon^n\}| 2^{-n\{I(X_1,X_2;Y|H_1,H_2)-7\epsilon\}}$$
$$\leq |\{\hat{w}_1^n\}|.|\{\hat{w}_2^n\}|.Pr\{(\hat{w}_1^n,\hat{w}_2^n)\in T_\epsilon^n\} .2^{-n\{I(X_1,X_2;Y|H_1,H_2)}$$
$$\leq 2^{n\{I(U_1;W_1)+I(U_2;W_2)+2\epsilon\}} 2^{-n\{I(W_1;W_2)-2\epsilon\}}2^{-n\{I(X_1,X_2;Y|H_1,H_2)-7\epsilon\}}$$
$$= 2^{n\{I(U_1,U_2;W_1,W_2)\}}2^{-n\{I(X_1,X_2;Y|H_1,H_2)-13\epsilon\}}.$$

The RHS of the above inequality tends to zero if $I(U_1,U_2;W_1,W_2) < I(X_1,X_2;Y|H_1,H_2)$.

Thus as $n\to\infty$, with probability tending to 1, the decoder finds the correct sequence (W_1^n,W_2^n) which is jointly weakly ϵ-typical with (U_1^n,U_2^n).

The fact that (W_1^n,W_2^n) are weakly ϵ-typical with (U_1^n,U_2^n) does not guarantee that $f_D^n(W_1^n,W_2^n)$ will satisfy the distortions D_1,D_2. For this, one needs that (W_1^n,W_2^n) are distortion-ϵ-weakly typical ([4]) with (U_1^n,U_2^n). Let $T_{D,\epsilon}^n$ denote the set of distortion typical sequences ([4]). Then by strong law of large numbers $P(T_{D,\epsilon}^n|T_\epsilon^n)\to 1$ as $n\to\infty$. Thus the distortion constraints are also satisfied by (W_1^n,W_2^n) obtained above with a probability tending to 1 as $n\to\infty$. Therefore, if distortion measure d is bounded $\lim_{n\to\infty} E[d(U_i^n,\hat{U}_i^n)]\leq D_i+\epsilon\ i=1,2.$

If there exist u_i^* such that $E[d_i(U_i,u_i^*)]<\infty,\ i=1,2,$ then the result extends to unbounded distortion measures also as follows. Whenever the decoded (W_1^n,W_2^n) are not in the distortion typical set then we estimate $(\hat{U}_1^n,\hat{U}_2^n)$ as (u_1^{*n},u_2^{*n}). Then for $i=1,2$,

$$E[d_i(U_i^n,\hat{U}_i^n)]\leq D_i+\epsilon+E[d(U_i^n,u_i^{*n})\mathbf{1}_{\{(T_{D,\epsilon}^n)^c\}}]. \quad (11)$$

Since $E[d(U_i^n,u_i^{*n})]<\infty$ and $P[(T_{D,\epsilon}^n)^c]\to 0$ as $n\to\infty$, the last term of (11) goes to zero as $n\to\infty$.

The above proof also hold for continuous sources and continuous channels. The Markov lemma and weak typical decoding, the devices used to prove the theorem continue to hold and the proof extends with $E[X_i^2]\leq \overline{P}_i\ i=1,2.$

References

[1] R. Ahlswede and T. Han. On source coding with side information via a multiple access channel and related problems in information theory. *IEEE Trans. Inform. Theory*, IT-29(3):396–411, May 1983.

[2] E. Biglieri, J. Proakis, and S. Shamai. Fading channels: Information theoretic and communication aspects. *IEEE Trans. Inform. Theory*, 44(6):2619–2692, Oct. 1998.

[3] T. M. Cover, A. E. Gamal, and M. Salehi. Multiple access channels with arbitrarily correlated sources. *IEEE Trans. Inform. Theory*, IT -26(6):648–657, Nov 1980.

[4] T. M. Cover and J. A. Thomas. *Elements of Information theory*. Wiley Series in Telecommunication, N.Y., 2004.

[5] M. Gastpar and M. Vetterli. Power spatio-temporal bandwidth and distortion in large sensor networks. *IEEE JSAC*, 23(4):745–754, April 2005.

[6] A. J. Goldsmith and P. P. Varaiya. Capacity of fading channels with channel side information. *IEEE Trans. Inform. Theory*, 43(6):1986–1992, Nov 1997.

[7] P. Ishwar, R. Puri, K. Ramchandran, and S. S. Pradhan. On rate constrained distributed estimation in unreliable sensor networks. *IEEE JSAC*, 23(4):765–775, April 2005.

[8] R. Knopp and P. A. Humblet. Information capacity and power control in single-cell multiuser communication. *Proc. Int. Conf. on Communication, ICC'95, Seattle,WA*, pages 331–335, June 1995.

[9] R. Knopp and P. A. Humblet. Multiple-accessing over frequency-selective fading channels. *6 th IEEE Int. Symp. on Personal Indoor and Mobile Radio Communication, PIMRC'95, Toronto, Canada*, pages 1326–1331, sept 1995.

[10] A. Lapidoth and S. Tinguely. Sending a bi- variate Gaussian source over a Gaussian MAC. *IEEE ISIT 06*, 2006.

[11] N. Liu and S. Ulukus. Capacity region and optimum power control stategies for fading Gaussian multiple access channels with common data. *IEEE Trans. communication*, 54(10):1815–1826, Oct. 2006.

[12] Y. Oohama. Gaussian multiterminal source coding. *IEEE Trans. Inform. Theory*, IT -43(6):1912–1923, Nov. 1997.

[13] R. Rajesh and V. Sharma. Source channel coding for Gaussian sources over a Gaussian multiple access channel. *Proc. 45 Allerton conference on Computing Control and Communication*, 2007.

[14] R. Rajesh and V. Sharma. Distributed joint source-channel coding on a multiple access channel with side information,. *Technical Report,Available: Arxiv, http://arxiv.org/PS_cache/arxiv/pdf/0803/0803.1445v1.pdf*, 2008.

[15] R. Rajesh, V. K. Varsheneya, and V. Sharma. Distributed joint source-channel coding on a multiple access channel with side information. *In Proc. IEEE ISIT 2008, Toronto, Canada.*

[16] S. Ray, M. Medard, M. Effros, and R. Kotter. On separation for multiple access channels. *Proc. IEEE Inform. Theory Workshop*, 2006.

[17] S. Shamai and A. D. Wyner. Information-theoretic considerations for symmetric cellular, multiple-access fading channels part I. *IEEE Trans. Inform. Theory*, 43(6):1877–1893, Nov 1997.

[18] D. Tse and S. V. Hanly. Multiaccess fading channels-part I: polymatroid structure, optimal resource allocation and throughput capacities. *IEEE Trans. Inform. Theory*, 44(7):2796–2815, Nov 1998.

[19] D. Tse and S. V. Hanly. Multiaccess fading channels-part ii: Delay-limited capacities. *IEEE Trans. Inform. Theory*, 44(7):2816–2831, Nov 1998.

[20] V. K. Varsheneya and V. Sharma. Distributed coding for multiple access communication with side information. *Proc. IEEE Wireless Communication and Networking Conference (WCNC)*, April 2006.

[21] A. B. Wagner, S. Tavildar, and P. Viswanath. The rate region of the quadratic Gaussian two terminal source coding problem. *IEEE Trans. Inform. Theory*, 54(5):1938–1961, May 2008.