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Performance Analysis of a Cooperative System with Rateless Codes and Buffered Relays

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***Abstract*—In a cooperative relay-assisted communication system that uses rateless codes, packets get transmitted from a source to a destination at a rate that depends on instantaneous channel states of the wireless links between nodes. When multiple relays are present, the relay with the highest channel gain to the source is the first to successfully decode a packet from the source and forward it to the destination. Thus, the unique properties of rateless codes ensure that both rate adaptation and relay selection occur without the transmitting source or relays acquiring instantaneous channel knowledge. In this paper, we show that in such cooperative systems, buffering packets at relays significantly increases throughput. We develop a novel analysis of these systems that combines the communication-theoretic aspects of cooperation over fading channels with the queuing-theoretic aspects associated with buffering. Closed-form expressions are derived for the throughput and end-to-end delay for the general case in which the channels between various nodes are not statistically identical. Corresponding results are also derived for benchmark systems that either do not exploit spatial diversity or do not buffer packets. Altogether, our results show that buffering – a capability that will be commonly available in practical deployments of relays – amplifies the benefits of cooperation.**

***Index Terms*—Cooperative communications, relay selection, queuing, rateless codes, fading channels, round robin, delay, throughput.**

I. Introduction

COOPERATIVE communications exploits the broadcast nature of the wireless channel and the spatial diversity offered by having geographically separated relays in the network [1]–[6]. When many relays are present in the system, selection based cooperation selects the relay with the most favorable instantaneous channel gain to forward information to the destination. Selection is practically appealing because tight symbol-level synchronization is not required among the relays. It fully harnesses the spatial diversity provided by multiple relays in the network since the selection of the best relay depends on the instantaneous channel gains of *all* the relays in the system [7]–[15].

Recently, the use of rateless codes has been shown to be well suited for cooperative relay networks [16]–[20]. Cooperation using rateless codes is attractive because the realized data transmission rate is automatically a function of the instantaneous channel state, without the transmitting source or relays knowing it. This is an important advantage because the time and bandwidth cost of acquiring channel knowledge at geographically separated nodes can be significant [21]. *Unlike conventional fixed-rate codes, which generate a finite number of parity bits, rateless codes generate a potentially infinite number of parity bits, which are transmitted until an acknowledgment (ACK) is received from the recipient [22]–[27].* The rateless codes are also efficient because the receiver can successfully decode the the packet once the mutual information that it has accumulated marginally exceeds the entropy of the source information.

In a cooperative multi-relay system in which the source transmits using rateless codes, the relay with the best source-relay (SR) channel will be the first to decode the source's packet as it accumulates mutual information at the fastest rate. Thus, relay selection automatically occurs without channel state information (CSI) at the source. This fact was exploited in [18], [19], which considered the following cooperation protocol: The source broadcasts its rateless coded bits until the $L \geq 1$ relays successfully decode the packet and send ACKs back to the source. Thereafter, these relays reencode the packet using rateless codes and transmit it to the destination simultaneously over non-interfering channels. The other relays discard their received and processed signals of this packet. The destination then receives the packet from the L transmitting relays, and decodes it. Once the destination has decoded the packet, the source starts transmitting the next packet.

In this paper, we consider the scenario where relays are capable of buffering packets that they have decoded in their queues. This capability is already a common feature in practical deployments of communicating nodes. It has also been used to improve spectrum access opportunities for cognitive radios in [28]. In our system, the source starts transmitting its next packet as soon as a relay has decoded it and stored

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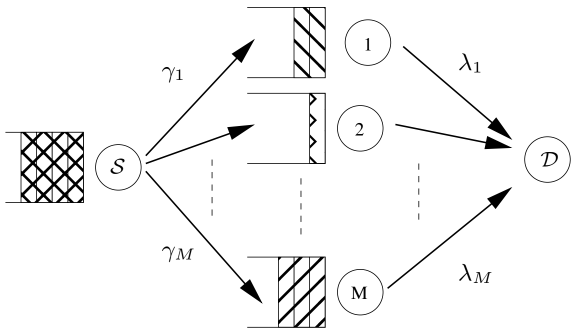

Fig. 1. Schematic of a queued cooperative relay network consisting of a source ($\mathcal{S}$), a destination ($\mathcal{D}$), and M relays ($\mathcal{R}_1, \ldots, \mathcal{R}_M$).

it in its queue for transmission to the destination. We show that buffering significantly enhances the throughput gains from cooperation at the expense of an increase in the average end-to-end (E-E) transit time through the network for a packet.

Our main contributions are the following. We develop a comprehensive analysis of a queued cooperative selection system that uses rateless codes and buffered relays, and verify it using extensive Monte Carlo simulations. The analysis leads to closed-form expressions for the average throughput and the average E-E transit time of a packet to reach the destination. It combines the communication-theoretic aspects of rateless coded cooperation over fading channels with the queuing-theoretic aspects associated with buffering at the relays. Expressions are derived for the general case in which the different SR and relay-destination (RD) channels are independent, but perhaps have different distributions. To help build intuition, we also analyze the symmetric case in which the SR channels are independent and identically distributed (i.i.d.) and so are the RD channels. To quantify the combined benefits of queuing and cooperation, we also analyze: (i) a baseline traditional queued relay system that does not exploit the spatial diversity benefits of relay selection, and (ii) a conventional cooperative system whose relays do not buffer packets.[1]

The outline of the paper is as follows. We describe the system model in Section II, and develop the analysis for the queued cooperative and non-cooperative schemes in Section III. Simulation results are presented in Section IV, and are followed by our conclusions in Section V. Several mathematical details are relegated to the Appendix.

II. System Model

As shown in Fig. 1, we consider a two-hop network in which the source, $\mathcal{S}$, has a continuous stream of packets, each B bits long, to transmit to the destination, $\mathcal{D}$, via M decode-and-forward relays, $\mathcal{R}_1, \ldots, \mathcal{R}_M$. Relay i's queue is denoted by $\mathcal{Q}_i$.

Channel Model: The signal transmitted by a source is received by multiple relays due to the broadcast nature of the wireless channel. The wireless channels between the different nodes are assumed to be frequency-flat channels. While the analysis is applicable, in general, to any fading distribution, the results are specialized for Rayleigh fading. The SR and RD channel gains for different relays are assumed to be mutually independent.

The source does not know the instantaneous channel gains of the various SR channels, and the relays do not know the instantaneous channel gains of their respective RD channels. But the transmitting source and the relays have knowledge of the statistics of the channel they are transmitting on. The channel gain of a link is assumed to be constant over the duration of transmission of the packet over the link; it changes to an independent value thereafter. To simplify the theoretical treatment, we ignore the direct source-destination link by assuming that it is blocked or too weak to be used [6], [11], [18], [30].

Source and Relay Transmission Protocol: The source and relays both use rateless codes to transmit their packets. In order for the various transmissions to be resolvable, different links use different orthogonal channels, each of bandwidth Ω Hz, and do not interfere with each other. This enables the source and relays to transmit in a decentralized manner because the allocation of channels to the source and relays only needs to be done once in the beginning [31]. When a relay accumulates sufficient mutual information, it is able to successfully decode the packet and it sends a short ACK back to the source.

The packet is then queued in the relay's buffer for transmission in a first come first serve fashion to the destination. For simplicity, the buffer lengths at the relays are assumed to be infinite. This is a realistic assumption for most buffer sizes seen in commercial products, and is commonly employed in the queuing literature. Note, however, that a larger buffer size increases the cost of a relay, especially a low cost one. Upon receiving an ACK, the source commences transmission of the next packet. The other relays discard whatever mutual information they have accumulated about the previous packet and start receiving the new packet from the source.[2] We focus on decode-and-forward relaying because each relay then only needs to store in its buffer the decoded B information bits of a packet.

The time taken by $\mathcal{R}_i$ to decode a packet transmitted by $\mathcal{S}$ is $(1+\delta)\frac{B}{\Omega \log_2(1+\gamma_i)} \triangleq \frac{\tilde{B}}{\log_e(1+\gamma_i)}$, where γ_i is the SNR of the link between $\mathcal{S}$ and $\mathcal{R}_i$, Ω is the bandwidth of the SR channel, $\delta \geq 0$ is the inefficiency of a practical rateless code, and $\tilde{B} = B(1+\delta)\log_e(2)/\Omega$. When the fading is Rayleigh, $\gamma_1, \ldots, \gamma_M$ are exponentially distributed random variables (RVs). Similarly, the SNRs of the RD links, denoted by λ_i for Relay i, are also exponential RVs.

The relays are assumed to be full-duplex, a capability that is pervasive, for example, in the frequency division duplexing mode of operation in cellular phones [32]. Therefore, it is

[1] Preliminary results were reported in a conference paper [29], which only analyzed the case where the SR and RD channels are i.i.d. This paper analyzes the general case where the various channels are not identical. Furthermore, even for the i.i.d. case, this paper contains the detailed proofs that were not given in [29].

[2] The ACKs are assumed to be sent on a separate common low-bandwidth channel that is error-free and incurs negligible delays. This is justifiable because an ACK is sent only once for every packet, and it contains just one bit of information. We also assume that a relay can ascertain that it has decoded the packet correctly, which can be accomplished by embedding cyclic redundancy check bits in the packet. A relay knows that $\mathcal{S}$ has started transmitting the next packet by either overhearing an ACK of another relay or by learning about it directly from $\mathcal{S}$. Since the fading distribution is continuous, with probability 1, no two relays decode the source's packet simultaneously. Therefore, we do not consider the minor impact of ACK collisions.

reasonable to assume it for relays as well. Full duplexing is practically achieved by using a duplexer and spacing the carrier frequencies of the SR channels and RD channels sufficiently apart [33].[3]

Channel Model and Its Implications: As mentioned, the channel is assumed to remain constant over the duration of a packet transmission and then changes to an independent value thereafter. A more realistic assumption would be a block fading model in which the channel remains constant over a fixed duration that is equal to the coherence time of the channel and changes to an independent value thereafter. However, the analysis of multi-relay systems with rateless codes becomes intractable. Therefore, the analysis is also intractable for the even more physically realistic Jake's fading model.

Fundamentally, the intractability occurs because the differences in times between successive packet transmissions by the source and the relays become correlated. Consequently, the packet arrival and departure processes make the buffer at each relay a $G/G/1$ queue [36], in which the inter-arrival times are correlated and so are the service times. Even good approximations for the throughput and E-E transit time of a $G/G/1$ queue have long remained an open problem.

The channel model used in this paper, which we shall refer to as the *per packet fading model*, helps us circumvent this critical bottleneck and enables an insightful analytical evaluation of the performance trade-offs of buffering at the relays. For the same reason, it has also been used in [16], [18], [19]. Even with this simplification, the analysis for our problem will turn out to be quite intricate since the buffers are $GI/GI/1$ queues [36], in which the inter-arrival times in any given relay's queue are i.i.d., but are correlated across different relay queues due to relay selection. The simplified channel model shows that the throughput gains from buffering and cooperation are substantial. In fact, as we shall see later, the per packet fading model underestimates these gains.

One implication of using the per packet fading model is the need for a time-out mechanism, in which the transmitting node times out if it does not receive an ACK within a duration t_{out}. This occurs when none of the receivers can decode the packet within time t_{out}. It ensures that the average transmission time is bounded. It is also motivated by a similar mechanism that is used in practical automatic repeat request (ARQ) and hybrid ARQ protocols [37]. As observed in [38], rateless codes, in effect, implement an efficient continuous incremental redundancy mechanism. Time-out can also be construed as a maximum decoding delay constraint. In the event of a time-out, the transmitting node can either drop the packet from its queue or retransmit it. Due to space constraints, we focus on packet dropping in this paper.

Possible Variants and Generalizations: Since our main goal is to investigate the impact of buffering on a cooperative system, we analyze the system described above as it has already been studied in the literature in the absence of buffering. Several variations in the cooperation protocol are possible. For example, the source can wait for multiple relays to acknowledge before starting the transmission of the next packet [18]. In general, the SR and RD links need not be orthogonal. In this context, an interesting problem is the analysis of a system that uses half-duplex relays, which cannot transmit and receive simultaneously in the same frequency band. Further, the selection of the relay can be made to depend on its RD channel gain [19], albeit with an increased protocol overhead.

A. Schemes Compared

We shall analyze and compare the throughputs and E-E transit times of the following two schemes:

1. Cooperative Relay Selection (RS): In this scheme, the source transmits a packet until *any one* of the relays decodes the packet. It is clear that the relay with the highest instantaneous SR link gain, which we call the *selected* relay, is the one to first decode the packet. Since the SR channel gains of all the relays together determine which relay gets selected, this scheme harnesses the spatial diversity afforded by multiple relays. The source transmits the next packet after it has received an ACK from the selected relay or it times out. The relay buffers the packet in its queue for transmission to the destination; the other relays discard the signals that they have received for this packet.

2. Non-cooperative Round-Robin Relay Selection (RR): We also consider a non-cooperative relay selection scheme that also uses rateless codes for transmission and buffers packets at the relays. In RR, the source *a priori* selects the relay in a round-robin (sequential) manner and transmits to it. Only the selected relay decodes the packet, stores it in its queue, and sends an ACK. As in RS, the source transmits the next packet thereafter.[4]

Comparing RS and RR is very instructive because it quantifies how cooperative diversity affects the queuing related behavior of the transmission scheme and vice versa. To better understand the impact of queuing, the performance of RS and RR when the relays do not buffer packets is also analyzed.

B. Notation

For an RV Y, $Y[k]$, $k = 1, 2, \ldots$, will denote an i.i.d. sequence with the distribution of Y. $\mathbf{E}[.]$ and $\text{Var}[.]$ shall denote the expectation and variance, respectively. Since $Y[k]$ is an i.i.d. sequence, we shall interchangeably use $\mathbf{E}\left[(Y[k])^m\right]$ and $\mathbf{E}[Y^m]$. The cumulative distribution function (CDF) of Y is denoted by $F_Y(.)$. The probability of an event A is denoted by $P(A)$. The cardinality of a set ω is denoted by $|\omega|$.

Let S^{SR} be the time taken to transmit a packet from the source; this includes packets that are dropped. Let S_i^{SR} denote the time taken by the source to transmit a packet conditioned on the event that $\mathcal{R}_i$ is the best relay. Let $A_i[k]$, $k = 1, 2, \ldots$,

[3]Our use of the term full-duplex is motivated by the terminology widely used in the cellular communications literature and wireless text books, e.g., [34]. Another interperation is an in-band full-duplex relay that transmits and receives on the same frequency band. We do not assume an in-band full-duplex relay as it requires advanced techniques to isolate transmit and receive circuitries and also signal cancellation techniques [35].

[4]An alternate non-cooperative scheme that resembles RS is probabilistic relay selection, in which the source decides a priori, for each packet, which relay to transmit to probabilistically. However, we do not analyze it because it has the same throughput as the RR scheme but with a provably larger average E-E transit time.

denote the successive inter-arrival times between packets that are successfully decoded by Relay i. Let $S_{\text{nd}}^{\text{SR}}$ denote the time taken by the source to transmit a packet given that it is not dropped, *i.e.*,

$$S_{\text{nd}}^{\text{SR}} = \left[S^{\text{SR}} | S^{\text{SR}} < t_{\text{out}}\right]. \quad (1)$$

The transmission time of a packet that is not dropped, given that $\mathcal{R}_i$ receives it, is denoted by $S_{\text{nd},i}^{\text{SR}}$. Similarly, S_i^{RD} is the time take by $\mathcal{R}_i$ to transmit its packet to $\mathcal{D}$; this includes packets that are dropped. And, $S_{\text{nd},i}^{\text{RD}}$ is the transmission time given that the packet is not dropped by $\mathcal{R}_i$. The probability that the source times out is denoted by $P_{\text{out}}^{\text{SR}}$. The probability that the source times out conditioned on $\mathcal{R}_i$ being the selected relay is denoted by $P_{\text{out},i}^{\text{SR}}$. Borrowing terminology from queuing theory, we shall interchangeably use the terms 'packet transmission time' and 'service time'. Let $\rho_i^{\text{SR}} = \mathbf{E}[\gamma_i]$ and $\rho_i^{\text{RD}} = \mathbf{E}[\lambda_i]$.

C. Relevant Queuing Theory Results

Our goal is to analyze the throughput and E-E transit time of the system. One key consequence of the system model is that the inter-arrival time of packets are i.i.d. with a *general* distribution, and so are the service times. Therefore, the queues that arise will be $GI/GI/1$ queues [36]. We summarize below some relevant results on such queues that will be used in our analysis.

Stability: By stable, we mean that the queue length and waiting time of an arbitrary packet in the queue have unique stationary distributions; starting from any initial condition, these processes converge weakly to the stationary distributions. A queue with inter-arrival time of packets given by the RV A and service time per packet given by the RV S is stable if and only if [36], [39]

$$\mathbf{E}[A] > \mathbf{E}[S]. \quad (2)$$

Waiting time: For a general $GI/GI/1$ queue, an exact expression for the mean waiting time is not available. However, an approximation for it is [40]

$$\mathbf{E}[W] \approx \frac{\eta g \mathbf{E}[S]\left(C_S^2 + C_A^2\right)}{2(1-\eta)}, \quad (3)$$

where $\eta = \frac{\mathbf{E}[S]}{\mathbf{E}[A]}$, $C_Y^2 = \frac{\text{Var}[Y]}{(\mathbf{E}[Y])^2}$, for $Y = A$ or S, and

$$g = \begin{cases} \exp\left(\frac{-2(1-\eta)(1-C_A^2)^2}{3\eta(C_A^2+C_S^2)}\right), & C_A^2 < 1 \\ \exp\left(-(1-\eta)\frac{C_A^2-1}{C_A^2+4C_S^2}\right), & C_A^2 \geq 1 \end{cases}.$$

It is known to be accurate under heavier traffic loads.

Other Performance Metrics: Since different packets are transmitted by different relays and with different transmission times, the packets need not be delivered in sequence to the destination. Consequently, a performance metric that is of interest to applications such as multi-media streaming is resequencing delay, which accounts for the delay incurred by an application in reordering the received packets. However, an analysis of the resequencing delay is well beyond the scope of this paper.

III. Analysis For Fading Channels

We now analyze the system model of Section II, in which the various links undergo Rayleigh fading. The analysis for all schemes proceeds as follows: we first analyze the SR link, followed by the queue at any relay, and then the RD link. We use these to derive exact closed-form expressions for the probability distributions and moments of the transmission times of the source and relays. These then lead to expressions for the performance metrics mentioned above. The only source of approximation is the computation of the average waiting time in (3). The results are also specialized for symmetric links case, in which $\rho_i^{\text{SR}} = \rho^{\text{SR}}$ and $\rho_i^{\text{RD}} = \rho^{\text{RD}}$, for all i. Note that the analysis makes no assumptions about the order in which the packets reach the destination.

A. Cooperative Relay Selection With Queues

In this case, the relay with the highest SR channel gain will receive the packet first.

1) Statistics of S_i^{SR}: The key results about the probability density function (PDF) of $S_{\text{nd},i}^{\text{SR}}$ and $P_{\text{out},i}^{\text{SR}}$ are captured in the following lemma.

Lemma 1: The probability, p_i, that $\mathcal{R}_i$ is selected equals

$$p_i = \frac{1}{\rho_i^{\text{SR}}} \sum_{k=1}^{2^{M-1}} (-1)^{|\omega_k|} a_{ik}, \quad (4)$$

where $a_{ik} \triangleq \left(\frac{1}{\rho_i^{\text{SR}}} + \sum_{j\in\omega_k} \frac{1}{\rho_j^{\text{SR}}}\right)^{-1}$, ω_k is the kth subset of the set $\{1,\ldots,M\} \setminus \{i\}$, and $1 \leq k \leq 2^{M-1}$.

The probability, $P_{\text{out},i}^{\text{SR}}$, that the source times out given that $\mathcal{R}_i$ is selected equals

$$P_{\text{out},i}^{\text{SR}} = \frac{1}{p_i \rho_i^{\text{SR}}} \sum_{k=1}^{2^{M-1}} (-1)^{|\omega_k|} a_{ik} \times \left(1 - \exp\left[\frac{1}{a_{ik}}\left(1 - e^{\tilde{B}/t_{\text{out}}}\right)\right]\right). \quad (5)$$

The PDF, $p_{S_{\text{nd},i}^{\text{SR}}}(x)$, of $S_{\text{nd},i}^{\text{SR}}$ is as follows: For $0 \leq x < t_{\text{out}}$,

$$p_{S_{\text{nd},i}^{\text{SR}}}(x) = \frac{1}{\left(1 - P_{\text{out},i}^{\text{SR}}\right)} \frac{\tilde{B}}{p_i \rho_i^{\text{SR}} x^2} \times \sum_{k=1}^{2^{M-1}} (-1)^{|\omega_k|} \exp\left(\frac{\tilde{B}}{x} + \frac{1 - e^{\tilde{B}/x}}{a_{ik}}\right), \quad (6)$$

and $p_{S_{\text{nd},i}^{\text{SR}}}(x) = 0$, otherwise.

Proof: The proof is given in Appendix A. ■

For the symmetric link case, the above expressions simplify considerably. Specifically, we get $p_i = \frac{1}{M}$ and

$$P_{\text{out},i}^{\text{SR}} = \left(1 - \exp\left[\frac{1}{\rho^{\text{SR}}}\left(1 - e^{\tilde{B}/t_{\text{out}}}\right)\right]\right)^M. \quad (7)$$

Further, for $0 \leq x < t_{\text{out}}$,

$$p_{S_{\text{nd},i}^{\text{SR}}}(x) = \frac{1}{\left(1 - P_{\text{out},i}^{\text{SR}}\right)} \frac{\tilde{B}M}{\rho^{\text{SR}} x^2} \exp\left[\frac{\tilde{B}}{x} - \frac{1}{\rho^{\text{SR}}}\left(e^{\tilde{B}/x} - 1\right)\right] \times \left(1 - \exp\left[\frac{1}{\rho^{\text{SR}}}\left(1 - e^{\tilde{B}/x}\right)\right]\right)^{M-1}, \quad (8)$$

and $p_{S^{\rm SR}_{{\rm nd},i}}(x) = 0$, otherwise.

From (6), the expressions for the moments of $S^{\rm SR}_{{\rm nd},i}$ are as follows.

Lemma 2: The first and second order moments of $S^{\rm SR}_{{\rm nd},i}$ are given by

$$\mathbf{E}\left[\left(S^{\rm SR}_{{\rm nd},i}\right)^m\right] = \frac{1}{\left(1-P^{\rm SR}_{{\rm out},i}\right)}\frac{\tilde{B}^m}{p_i\rho_i^{\rm SR}} \times \sum_{k=0}^{2^{M-1}}(-1)^{|\omega_k|}\psi_m\left(a_{ik}, \frac{\tilde{B}}{t_{\rm out}}\right), \quad m = 1, 2, \quad (9)$$

where, for $a > 0$ and $u > 0$,

$$\psi_m(a,u) \triangleq \int_u^\infty \frac{1}{x^m}\exp\left(x + \frac{1-e^x}{a}\right)dx. \quad (10)$$

Proof: The proof is given in Appendix B. ∎

From the law of total expectation, the moments of $S_i^{\rm SR}$ follow directly from (9), and are given by

$$\mathbf{E}\left[\left(S_i^{\rm SR}\right)^m\right] = (1-P^{\rm SR}_{{\rm out},i})\mathbf{E}\left[\left(S^{\rm SR}_{{\rm nd},i}\right)^m\right] + P^{\rm SR}_{{\rm out},i}t^m_{\rm out}. \quad (11)$$

For symmetric links, the moment expressions in (9) simplify to

$$\mathbf{E}\left[\left(S^{\rm SR}_{{\rm nd},i}\right)^m\right] = \frac{1}{\left(1-P^{\rm SR}_{{\rm out},i}\right)}\frac{\tilde{B}^m M}{\rho^{\rm SR}}\sum_{k=0}^{M-1}(-1)^k\binom{M-1}{k} \times \psi_m\left(\frac{\rho^{\rm SR}}{k+1}, \frac{\tilde{B}}{t_{\rm out}}\right), \quad m = 1, 2. \quad (12)$$

2) Statistics of Inter-arrival Time A_i: Recall that a transmitting node times out after time $t_{\rm out}$ in case it does not receive an acknowledgment. Thus, a packet might get dropped and not enter its selected relay's queue if it is not decoded within time $t_{\rm out}$. Therefore, to analyze the inter-arrival time at a relay's queue, we use extensively a renewal concept of a cycle [36], which is the interval between two successive instances in which a given relay is the selected relay. In general, for any fading channel, the inter-arrival time at Relay i with packet dropping is

$$A_i = S^{\rm SR}_{{\rm nd},i} + \sum_{l=1}^{c_i}\sum_{\substack{j=1\\(j\neq i)}}^{M}\sum_{k=1}^{N_j^{(i)}[l]} S_j^{\rm SR}[k] + (c_i - 1)t_{\rm out}, \quad (13)$$

where c_i is the number of cycles between two *successful* packet arrivals in $\mathcal{Q}_i$ and $N_j^{(i)}[m]$ is the number of times $\mathcal{R}_j$ is the best relay in the mth cycle.[5]

The above equation is illustrated in Fig. 2, and can be understood as follows. As mentioned, a cycle is the inter-arrival time between consecutive epochs in which $\mathcal{R}_i$ is the best relay. For $1 \leq m < c_i$, the mth cycle duration is

$$\sum_{\substack{j=1\\(j\neq i)}}^{M}\sum_{k=1}^{N_j^{(i)}[m]} S_j^{\rm SR}[k] + t_{\rm out},$$

[5]Strictly speaking, in (13), the variable k would range from $N_j^{(i)}[1]+\cdots+N_j^{(i)}[l-1]+1$ to $N_j^{(i)}[1]+\cdots+N_j^{(i)}[l-1]+N_j^{(i)}[l]$, instead of from 1 to $N_k^{(i)}[l]$. However, we do not show this to keep the expressions simple, and shall easily side step any ambiguities because $S_j^{\rm SR}[k]$ in different cycles are i.i.d.

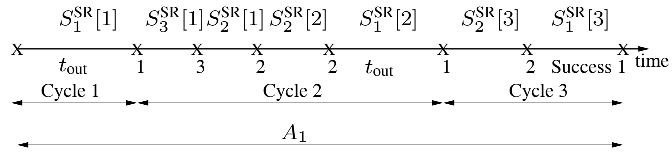

Fig. 2. Arrival process at Relay 1 in RS: The illustration shows an example with $M = 3$ relays and $c_1 = 3$ cycles. The crosses ($\times$) show epochs at which a packet is received successfully by a relay or dropped. In the first cycle, $N_2^{(1)}[1] = 0$ and $N_3^{(1)}[1] = 0$. In the second cycle, $N_2^{(1)}[2] = 2$ and $N_3^{(1)}[2] = 1$. In the third cycle, $N_2^{(1)}[3] = 1$ and $N_3^{(1)}[3] = 0$.

since the source times out in each of the first $c_i - 1$ cycles. The last (c_ith) cycle's duration equals

$$S^{\rm SR}_{{\rm nd},i} + \sum_{\substack{j=1\\(j\neq i)}}^{M}\sum_{k=1}^{N_j^{(i)}[c_i]} S_j^{\rm SR}[k],$$

since the packet is successfully received by $\mathcal{R}_i$ at the end of the cycle. This also implies that c_i is a geometric RV with probability

$$P(c_i = l) = (P^{\rm SR}_{{\rm out},i})^{l-1}(1-P^{\rm SR}_{{\rm out},i}), \quad l \geq 1. \quad (14)$$

The moments of $N_j^{(i)}$, which will be needed in the analysis, take the following simple form.

Lemma 3: The first and second order moments of $N_j^{(i)}$ are given by

$$\begin{aligned} \mathbf{E}\left[N_j^{(i)}\right] &= p_j/p_i, \\ \mathbf{E}\left[\left(N_j^{(i)}\right)^2\right] &= p_j(2p_j+p_i)/p_i^2, \\ \mathbf{E}\left[N_{j_1}[l]N_{j_2}[l]\right] &= 2p_{j_1}p_{j_2}/p_i^2, \quad j_1 \neq j_2, \quad \text{and} \\ \mathbf{E}\left[N_{j_1}[l_1]N_{j_2}[l_2]\right] &= p_{j_1}p_{j_2}/p_i^2, \quad l_1 \neq l_2. \end{aligned} \quad (15)$$

Proof: The proof is given in Appendix C. ∎

For the symmetric case, the results in Lemma 3 simplify to $\mathbf{E}\left[N_j^{(i)}\right] = 1$, $\mathbf{E}\left[\left(N_j^{(i)}\right)^2\right] = 3$, $\mathbf{E}\left[N_{j_1}^{(i)}[l]N_{j_2}^{(i)}[l]\right] = 2$, for $j_1 \neq j_2$, and $\mathbf{E}\left[N_{j_1}^{(i)}[l_1]N_{j_2}^{(i)}[l_2]\right] = 1$, for $l_1 \neq l_2$. *A key point to note is that within the lth cycle, $N_{j_1}^{(i)}[l]$ and $N_{j_2}^{(i)}[l]$ are correlated even when $j_1 \neq j_2$.*

The closed-form expressions for the mean and variance of A_i for RS are then as follows.

Lemma 4: The mean inter-arrival time of packets at $\mathcal{R}_i$ for RS is given by

$$\mathbf{E}[A_i] = \frac{1}{1-P^{\rm SR}_{{\rm out},i}}\sum_{j=1}^{M}\frac{p_j}{p_i}\mathbf{E}\left[S_j^{\rm SR}\right], \quad (16)$$

and the variance is given by

$$\mathrm{Var}[A_i] = \mathrm{Var}\left[S^{\rm SR}_{{\rm nd},i}\right] + \frac{1}{1-P^{\rm SR}_{{\rm out},i}}\sum_{\substack{j=1\\(j\neq i)}}^{M}\frac{p_j}{p_i}\mathbf{E}\left[\left(S_j^{\rm SR}\right)^2\right] + \frac{1}{1-P^{\rm SR}_{{\rm out},i}}\sum_{\substack{j=1\\(j\neq i)}}^{M}\frac{p_j^2}{p_i^2}\mathbf{E}^2\left[S_j^{\rm SR}\right]$$

$$+\frac{1}{1-P_{\text{out},i}^{\text{SR}}}\sum_{\substack{j_1=1\\(j_1\neq i)}}^{M}\sum_{\substack{j_2=1\\(j_2\neq j_1,i)}}^{M}\frac{p_{j_1}p_{j_2}}{p_i^2}\mathbf{E}\left[S_{j_1}^{\text{SR}}\right]\mathbf{E}\left[S_{j_2}^{\text{SR}}\right]$$
$$+\frac{P_{\text{out},i}^{\text{SR}}}{(1-P_{\text{out},i}^{\text{SR}})^2}\left[t_{\text{out}}+\sum_{\substack{j=1\\(j\neq i)}}^{M}\frac{p_j}{p_i}\mathbf{E}\left[S_j^{\text{SR}}\right]\right]^2. \quad (17)$$

Here, $\mathbf{E}\left[S_i^{\text{SR}}\right]$, $\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right]$, $\mathbf{E}\left[\left(S_i^{\text{SR}}\right)^2\right]$, and $\text{Var}\left[S_{\text{nd},i}^{\text{SR}}\right]$ are given by Lemma 2 and (11).

Proof: The proof is given in Appendix D. ■

For the symmetric case, the expressions simplify to

$$\mathbf{E}\left[A_i\right]=\frac{M\mathbf{E}\left[S_i^{\text{SR}}\right]}{1-P_{\text{out},i}^{\text{SR}}}, \quad (18)$$

and

$$\text{Var}\left[A_i\right]=\text{Var}\left[S_{\text{nd},i}^{\text{SR}}\right]+\frac{(M-1)^2}{1-P_{\text{out},i}^{\text{SR}}}\mathbf{E}^2\left[S_i^{\text{SR}}\right]$$
$$+\frac{M-1}{1-P_{\text{out},i}^{\text{SR}}}\mathbf{E}\left[\left(S_i^{\text{SR}}\right)^2\right]$$
$$+\frac{P_{\text{out},i}^{\text{SR}}}{\left(1-P_{\text{out},i}^{\text{SR}}\right)^2}\left(t_{\text{out}}+(M-1)\mathbf{E}\left[S_i^{\text{SR}}\right]\right)^2. \quad (19)$$

3) Statistics of Transmission Times at $\mathcal{R}_i$: The packet transmission times at $\mathcal{R}_i$ are an i.i.d. sequence. Similar to the SR link analysis, the PDF, $p_{S_{\text{nd},i}^{\text{RD}}}(s)$, of $S_{\text{nd},i}^{\text{RD}}$ is given as follows. For $0\le s<t_{\text{out}}$,

$$p_{S_{\text{nd},i}^{\text{RD}}}(s)=\frac{\tilde{B}}{\rho_i^{\text{RD}}s^2(1-P_{\text{out},i}^{\text{RD}})}\times\exp\left[\frac{\tilde{B}}{s}-\frac{1}{\rho_i^{\text{RD}}}\left(e^{\tilde{B}/s}-1\right)\right], \quad (20)$$

and $p_{S_{\text{nd},i}^{\text{RD}}}(s)=0$, otherwise. Here, $P_{\text{out},i}^{\text{RD}}$ is the probability that $\mathcal{R}_i$ times out. Since $\int_0^{\infty}p_{S_{\text{nd},i}^{\text{RD}}}(s)\,ds=1$, we get

$$P_{\text{out},i}^{\text{RD}}=1-\exp\left[-\frac{1}{\rho_i^{\text{RD}}}\left(e^{\tilde{B}/t_{\text{out}}}-1\right)\right]. \quad (21)$$

Finally, from (20) and (21), the first and second moments of S_i^{RD} are given in closed-form as

$$\mathbf{E}\left[\left(S_i^{\text{RD}}\right)^m\right]=P_{\text{out},i}^{\text{RD}}t_{\text{out}}^m+\frac{\tilde{B}^m}{\rho_i^{\text{RD}}}\psi_m\left(\rho_i^{\text{RD}},\frac{\tilde{B}}{t_{\text{out}}}\right). \quad (22)$$

4) Performance Measures: We are now ready to state the main results about the performance of RS.

Theorem *1:* The queue of Relay i in relay selection is stable if and only if

$$\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right]+\frac{P_{\text{out},i}^{\text{SR}}}{1-P_{\text{out},i}^{\text{SR}}}t_{\text{out}}+\frac{1}{(1-P_{\text{out},i}^{\text{SR}})}\sum_{\substack{j=1\\(j\neq i)}}^{M}\frac{p_j}{p_i}\mathbf{E}\left[S_j^{\text{SR}}\right]$$
$$>P_{\text{out},i}^{\text{RD}}t_{\text{out}}+\frac{\tilde{B}}{\rho_i^{\text{RD}}}\psi_1\left(\rho_i^{\text{RD}},\frac{\tilde{B}}{t_{\text{out}}}\right). \quad (23)$$

Proof: $\mathcal{Q}_i$ is a $GI/GI/1$ queue because the inter-arrival times are i.i.d., the packet transmission times are i.i.d., and the two are independent of each other. The desired result follows by substituting in (2) the results of Lemma 4 and (22). ■

For the symmetric case, the stability condition simplifies considerably to

$$\frac{1}{\left(1-P_{\text{out}}^{\text{SR}}\right)}\frac{\tilde{B}M^2}{\rho^{\text{SR}}}\sum_{k=0}^{M-1}(-1)^k\binom{M-1}{k}\psi_1\left(\frac{\rho^{\text{SR}}}{k+1},\frac{\tilde{B}}{t_{\text{out}}}\right)$$
$$+\frac{MP_{\text{out}}^{\text{SR}}}{1-P_{\text{out}}^{\text{SR}}}t_{\text{out}}>P_{\text{out}}^{\text{RD}}t_{\text{out}}+\frac{\tilde{B}}{\rho^{\text{RD}}}\psi_1\left(\rho^{\text{RD}},\frac{\tilde{B}}{t_{\text{out}}}\right). \quad (24)$$

Theorem *2:* The throughput of relay selection in both the stable and unstable regimes equals

$$\Lambda=\sum_{i=1}^{M}\frac{1-P_{\text{out},i}^{\text{RD}}}{\max\left(\mathbf{E}\left[S_i^{\text{RD}}\right],\mathbf{E}\left[A_i\right]\right)}\quad\text{packets/sec.} \quad (25)$$

Proof: The proof is given in Appendix E. ■

For the symmetric case, the throughput expression reduces to the following simple and intuitive form:

$$\Lambda=\frac{M\left(1-P_{\text{out},i}^{\text{RD}}\right)}{\max\left(\mathbf{E}\left[S_i^{\text{RD}}\right],\mathbf{E}\left[A_i\right]\right)}, \quad (26)$$

where i indexes any relay.

Finally, the average E-E transit time, ξ, for a packet that the destination decodes is given as follows.

Theorem *3:*

$$\xi=\left(\sum_{i=1}^{M}\left(\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right]+\mathbf{E}\left[W_i\right]+\mathbf{E}\left[S_{\text{nd},i}^{\text{RD}}\right]\right)\times\left(1-P_{\text{out},i}^{\text{SR}}\right)\left(1-P_{\text{out},i}^{\text{RD}}\right)p_i\right)\times\frac{1}{\sum_{i=1}^{M}\left(1-P_{\text{out},i}^{\text{SR}}\right)\left(1-P_{\text{out},i}^{\text{RD}}\right)p_i}, \quad (27)$$

where W_i is the waiting time of a packet in $\mathcal{Q}_i$.

Proof: The proof is relegated to Appendix F. ■

Note that in (27) $\mathbf{E}\left[W_i\right]$ is obtained by substituting the results from Lemma 4 and (22) in (3). For the symmetric case, (27) simplifies to

$$\xi=\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right]+\mathbf{E}\left[W_i\right]+\mathbf{E}\left[S_{\text{nd},i}^{\text{RD}}\right], \quad (28)$$

where i indexes any relay.

B. Non-cooperative Round-Robin Relay Selection (RR)

In the non-cooperative RR system, the source itself decides which relay to transmit to *before* transmitting a packet. It selects the relays in a sequential manner.

1) Statistics of S^{SR}: The closed-form expressions for $P_{\text{out},i}^{\text{SR}}$ and $p_{S_{\text{nd},i}^{\text{SR}}}(s)$ are as follows.

Lemma 5: The source time-out probability is

$$P_{\text{out},i}^{\text{SR}}=1-\exp\left[-\frac{1}{\rho_i^{\text{SR}}}\left(e^{\tilde{B}/t_{\text{out}}}-1\right)\right]. \quad (29)$$

The PDF, $p_{S_{\text{nd},i}^{\text{SR}}}(s)$, of $S_{\text{nd},i}^{\text{SR}}$, the time taken to transmit a packet to $\mathcal{R}_i$ given that the packet is not dropped, is as follows. For

$0 \leq s < t_{\text{out}}$,

$$p_{S_{\text{nd},i}^{\text{SR}}}(s) = \frac{1}{\left(1-P_{\text{out},i}^{\text{SR}}\right)} \frac{\tilde{B}}{\rho_i^{\text{SR}} s^2} \times \exp\left[\frac{\tilde{B}}{s} - \frac{1}{\rho_i^{\text{SR}}}\left(e^{\tilde{B}/s}-1\right)\right], \quad (30)$$

and $p_{S_{\text{nd},i}^{\text{SR}}}(s) = 0$, otherwise.

Proof: The proof is along the lines of Appendix A, and is skipped to conserve space. ■

In effect, the results in Lemma 5 correspond to $M = 1$ in Lemma 1 since RR does not consider the SR channel condition. Therefore, the expressions for the moments of $S_{\text{nd},i}^{\text{SR}}$ are as follows.

Lemma 6: The first two moments of $S_{\text{nd},i}^{\text{SR}}$ are given by

$$\mathbf{E}\left[\left(S_{\text{nd},i}^{\text{SR}}\right)^m\right] = \frac{1}{\left(1-P_{\text{out},i}^{\text{SR}}\right)} \frac{\tilde{B}^m}{\rho_i^{\text{SR}}} \psi_m\left(\rho_i^{\text{SR}}, \frac{\tilde{B}}{t_{\text{out}}}\right), \; m = 1, 2. \quad (31)$$

Proof: Starting from the expression for $p_{S_{\text{nd},i}^{\text{SR}}}(s)$ in (30), the derivation is along lines similar to that for RS in Appendix B. We do not elaborate further in order to conserve space. ■

The expressions for $\mathbf{E}\left[S_i^{\text{SR}}\right]$ and $\mathbf{E}\left[\left(S_i^{\text{SR}}\right)^2\right]$ in terms of $\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right]$ and $\mathbf{E}\left[\left(S_{\text{nd},i}^{\text{SR}}\right)^2\right]$ then follow from (11).

2) Statistics of Inter-Arrival Time, A_i: The inter-arrival time expression for RR is similar to that for RS, except for one simplification – *in RR, a relay is selected exactly once in every cycle.* Therefore,

$$A_i = S_{\text{nd},i}^{\text{SR}} + \sum_{l=1}^{c_i} \sum_{\substack{j=1 \\ (j \neq i)}}^{M} S_j^{\text{SR}}[l] + (c_i - 1)t_{\text{out}}. \quad (32)$$

As before, $P(c_i = l) = \left(P_{\text{out},i}^{\text{SR}}\right)^{l-1}(1-P_{\text{out},i}^{\text{SR}})$, for $l \geq 1$.

Lemma 7: The mean of A_i is given by

$$\mathbf{E}[A_i] = \frac{1}{1-P_{\text{out},i}^{\text{SR}}} \sum_{j=1}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right], \quad (33)$$

and the variance of A_i is given by

$$\text{Var}[A_i] = \left[t_{\text{out}} + \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right]\right]^2 \frac{P_{\text{out},i}^{\text{SR}}}{\left(1-P_{\text{out},i}^{\text{SR}}\right)^2} + \frac{1}{1-P_{\text{out},i}^{\text{SR}}} \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \text{Var}\left[S_j^{\text{SR}}\right] + \text{Var}\left[S_{\text{nd},i}^{\text{SR}}\right]. \quad (34)$$

Proof: The proof is given in Appendix G. ■

For the symmetric case, for any Relay i, the expressions simplify to

$$\mathbf{E}[A_i] = \frac{M\mathbf{E}\left[S_i^{\text{SR}}\right]}{1-P_{\text{out},i}^{\text{SR}}}, \quad (35)$$

and

$$\text{Var}[A_i] = \text{Var}\left[S_{\text{nd},i}^{\text{SR}}\right] + \frac{M-1}{1-P_{\text{out},i}^{\text{SR}}}\text{Var}\left[S_i^{\text{SR}}\right] + \frac{P_{\text{out},i}^{\text{SR}}}{\left(1-P_{\text{out},i}^{\text{SR}}\right)^2}\left(t_{\text{out}} + (M-1)\mathbf{E}\left[S_i^{\text{SR}}\right]\right)^2, \quad (36)$$

where i indexes any relay.

3) Statistics of Transmission Time of $\mathcal{R}_i$: Clearly, the PDF of the relay transmission time is the same as that for RS since the relay transmits in the same manner to the destination. Its first and second moments are, therefore, given by (22). The probability, $P_{\text{out},i}^{\text{RD}}$, that Relay i times out is given by (21).

4) Performance Measures: Putting the above results together leads to the following complete characterization of the stability, throughput, and E-E transit time of RR.

Theorem *4:* The queue of Relay i in RR is stable if and only if

$$P_{\text{out},i}^{\text{RD}} t_{\text{out}} + \frac{\tilde{B}}{\rho_i^{\text{RD}}}\psi_1\left(\rho_i^{\text{RD}}, \frac{\tilde{B}}{t_{\text{out}}}\right) < \mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right] + \frac{1}{1-P_{\text{out},i}^{\text{SR}}} \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right] + \frac{P_{\text{out},i}^{\text{SR}}}{1-P_{\text{out},i}^{\text{SR}}} t_{\text{out}}. \quad (37)$$

Proof: For RR, $\mathcal{Q}_i$ is a $GI/GI/1$ queue because the inter-arrival times and the packet transmission times at each relay are i.i.d. and independent of each other [36]. The desired result follows by substituting (22) and (33) in (2). ■

The average throughput expression is again given by Theorem 2. Since the queues are $GI/GI/1$, the results for the average waiting time in $\mathcal{Q}_i$ and the average E-E transmit time follow by substituting the results from Lemma 7 and (22) in (3). The corresponding expressions for the symmetric case also follow along the lines for RS, and are not repeated here to conserve space.

C. Unbuffered Case: Both Cooperative (RS) and Non-cooperative (RR) Selection Schemes

Lemma 8: The average throughput (in packets/sec) when the relays are unbuffered equals

$$\Lambda = \frac{\sum_{i=1}^{M} p_i\left(1-P_{\text{out},i}^{\text{SR}}\right)\left(1-P_{\text{out},i}^{\text{RD}}\right)}{\sum_{i=1}^{M} p_i\left(1-P_{\text{out},i}^{\text{SR}}\right)\left(\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right] + \mathbf{E}\left[S_i^{\text{RD}}\right]\right) + p_i P_{\text{out},i}^{\text{SR}} t_{\text{out}}}. \quad (38)$$

Proof: The result follows because a fraction $P_{\text{out},i}^{\text{SR}}$ of the packets transmitted to $\mathcal{R}_i$ get dropped by the source. And, a fraction $P_{\text{out},i}^{\text{RD}}$ of the packets transmitted by $\mathcal{R}_i$ do not reach the destination. ■

Depending on the relay selection scheme under consideration (RS or RR), the corresponding values for $P_{\text{out},i}^{\text{SR}}$, p_i, $\mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right]$, and $\mathbf{E}\left[S_i^{\text{SR}}\right]$ are substituted in (38).

The expression for the E-E transit time of the unbuffered case is given by Theorem 3, with $\mathbf{E}[W_i]$ set to 0. For the symmetric case, it simplifies to the following intuitive form:

$$\xi = \mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right] + \mathbf{E}\left[S_{\text{nd},i}^{\text{RD}}\right]. \quad (39)$$

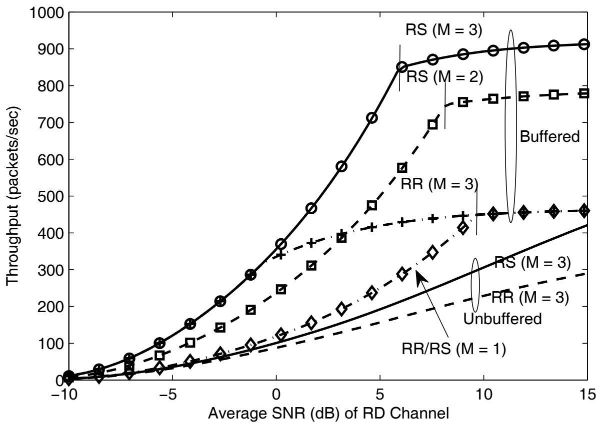

Fig. 3. Symmetric channels case: Throughput and stability regions for RS and RR for different numbers of buffered/unbuffered relays. Stability region boundaries for the buffered case are indicated using short vertical lines.

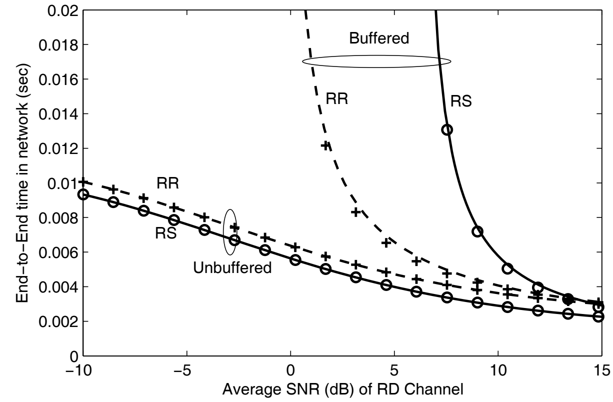

Fig. 4. Symmetric channels case: End-to-end transit time in the cooperative network for RS and RR with buffered and unbuffered relays ($M = 3$ relays).

IV. Simulation Results

We first study the symmetric channels case to gain insights into the behavior of the system. We then present results for the general case. *In Figures 3 to 7, analysis results are plotted using lines (– and -.-) and Monte Carlo simulation results that use up to* 10^7 *samples are shown using markers (◇, □, ○, etc.).*

A. Symmetric Relay Channels

The following system parameters are used: bandwidth $\Omega = 1$ MHz, $B = 4096$ bits per packet, $\delta = 0$ (ideal rateless code), $\rho^{\text{SR}} = 10$ dB, and $t_{\text{out}} = 10$ msec.

Figure 3 shows the throughput and the stability region for RS and RR as a function of the average RD link SNR for different numbers of relays. Notice that the simulation and analytical results match well in all cases. Both the throughput and stability region increase as the number of relays increases and as the average RD link SNR improves. For example, RS is stable for $\rho^{\text{RD}} \geq 9.7$ dB for $M = 1$ and for $\rho^{\text{RD}} \geq 6.0$ dB for $M = 3$. Since RR loads the relay queues less, its stability region is larger ($\rho^{\text{RD}} \geq 0$ dB for $M = 3$). Compared to the unbuffered relays case, in which the relay queues are always stable, queuing combined with RS significantly increases the throughput. For example, for $M = 3$ relays, throughput improves by 192% at $\rho^{\text{RD}} = 10$ dB and by 143% at $\rho^{\text{RD}} = 13$ dB.

The E-E transit times are compared in Fig. 4 as a function of the RD link SNR, ρ^{RD}, for $M = 3$. The simulation and analytical results again match each other well. For the same ρ^{RD}, RS has a higher E-E transit time than RR in the buffered case. This is because, in the presence of buffering, the higher SR link throughput of RS loads the relay queues more than in RR; this increases the waiting times at the relays. Interestingly, in line with the intuition behind communication-theoretic approaches [18], the trend is reversed when the relays do not buffer packets. Now RS has the smallest E-E transit time because it reduces the SR transmission time, and it does not affect the RD transmission time. As we saw in Section III-C, these two transmission times entirely determine the E-E transit time in the unbuffered case.

B. General Case: Asymmetric SR and RD Channels

We now focus on the impact of asymmetry among the various SR and RD links. It leads to one of the following two scenarios: (i) A relay has a strong SR link but a weak RD link, in which case the packets can quickly get backlogged in the relay's queue; (ii) Or, the relay has a weak SR link and a strong RD link, in which case the backlog is smaller. To illustrate this, we consider $M = 3$ relays and use the following parameter values: $\rho_1^{\text{SR}} = 10$ dB, $\rho_2^{\text{SR}} = 15$ dB, and $\rho_3^{\text{SR}} = 20$ dB. For the RD links, we maintain the same ratios between the average link gains as for the SR links, but allow for different permutations among them. The notation for permutations used in the figure is best explained by an example. In the permutation $[10\ 0\ 5]$, the ρ_1^{RD} and ρ_3^{RD} are 10 dB and 5 dB, respectively, more than $\rho_2^{\text{RD}} = 10$ dB.

As was done for the symmetric case, we plot in Figure 5 the total throughput as a function of the mean RD channel SNR (averaged over all three RD channels) for different permutations of the RD link SNRs. For RS, the highest throughput is achieved by the permutation $[0\ 5\ 10]$, in which the relay with a stronger SR link also has a stronger RD link. As expected intuitively, the worst throughput occurs for the permutation $[10\ 5\ 0]$, in which the weakest RD link is paired with the strongest SR link. On the other hand, for RR, the asymmetry does not affect the throughput. Asymmetry, however, does make the stability regions of the three relay queues different in both RS and RR. Figure 6, which zooms into the stable throughput regions, shows that the difference between the throughputs of the various permutations is smaller in the stability region. This happens because when all the 3 relays become stable, the RD link SNRs are already so high that the strongest SR channel itself becomes the bottleneck.

Figure 7 plots the E-E transit time as a function of the mean RD channel gain for RR and RS for the two permutations that are diametrically opposite to each other in performance. The other permutations are not shown to maintain clarity. We again see that the effect of asymmetry is manifested more for RS than RR. As in the symmetric case, the E-E transit time of RS exceeds that of RR because of the higher rate at which the source can push packets into the relay buffers in RS.

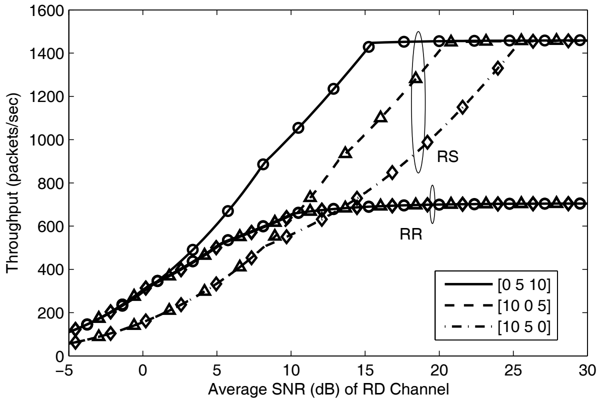

Fig. 5. General asymmetric channels case: Throughput vs. average SNR of RD links for RS and RR ($M = 3$ relays).

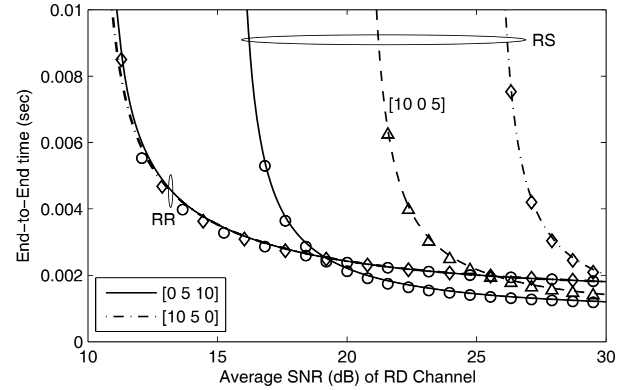

Fig. 7. General asymmetric channels case: End-to-end transit time vs. average SNR of RD links for RS and RR for different permutations of the RD link SNRs.

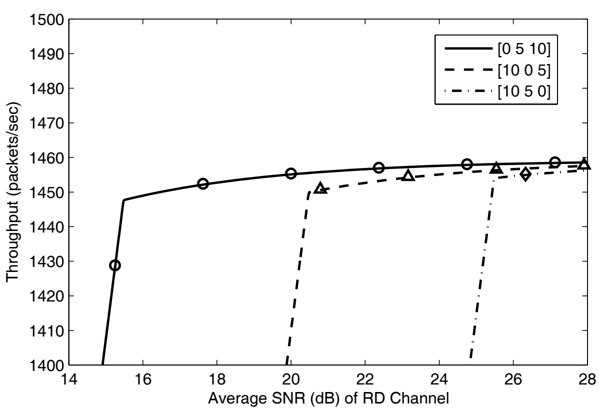

Fig. 6. General asymmetric channels case: Zoomed-in view of throughput vs. average SNR of RD link curves for RS for different permutations of the RD link SNRs.

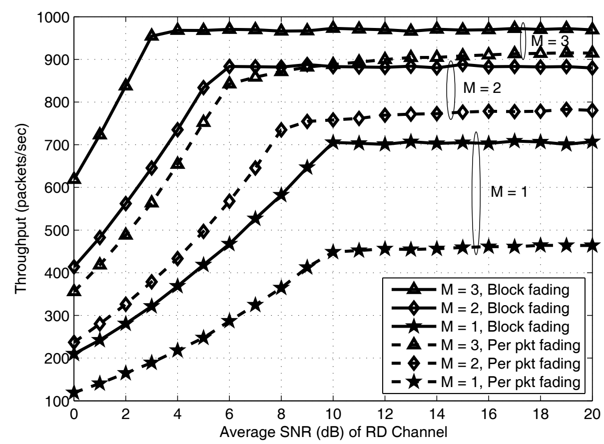

Fig. 8. Comparison of the throughput for the block fading channel model and the per packet fading model as a function of the number of relays.

C. Block Fading Channel Model

Figures 8 and 9 plot the throughput and E-E transit time for the per packet fading model, analyzed in the paper, and the block fading channel model, in which the channel remains constant for a fixed duration of 10 msec and changes to an independent value thereafter. All other parameters are the same as those used for Figures 3 and 4. We see that the throughput for the block fading model is greater than that for the per packet fading model. Thus, the per packet fading model, in fact, underestimates the gains from buffering packets at the cooperative relays.

V. Conclusions

We analyzed a cooperative system that uses rateless codes, in which the relays have queues and the wireless links undergo fading. Closed-form expressions were developed to characterize the overall performance of the system in terms of its throughput, end-to-end delay, and stability of relay queues. This was done for cooperative relay selection and also for traditional benchmark cases where either buffering or cooperative diversity was absent. We saw that buffering at relays amplifies the benefits of cooperation and significantly increases the throughput at the expense of a larger delay. We found that the throughput and stability region increase and the queuing delays decrease as the number of relays or the average RD link SNR increase. An increase in the number of relays yields benefits not only in the form of greater spatial diversity, but also helps reduce the traffic load at each relay. This, in turn, reduces the end-to-end delays. The traditional buffered relay models exploit only the latter aspect.

An interesting avenue for future research is to develop transmission policies at the source that modify the trade-off between throughput and delay or, in general, the trade-offs between throughput, delay, and energy consumed. Further, developing an analysis of the performance under the less pessimistic block fading channel model is a relevant and challenging problem.

Appendix

A. Proof of Lemma 1

We first consider the case where there is no time-out. Let κ_i^{SR} denote the time taken to transmit a packet to $\mathcal{R}_i$, if it were the only relay in the system. Then, $F_{\kappa_i^{\text{SR}}}(x) = P\left(\frac{\tilde{B}}{\log_e(1+\gamma_i)} < x\right) = P\left(\gamma_i > e^{\tilde{B}/x} - 1\right) = \exp\left(-\frac{1}{\rho_i^{\text{SR}}}\left(e^{\tilde{B}/x} - 1\right)\right)$, since γ_i is an exponential RV with

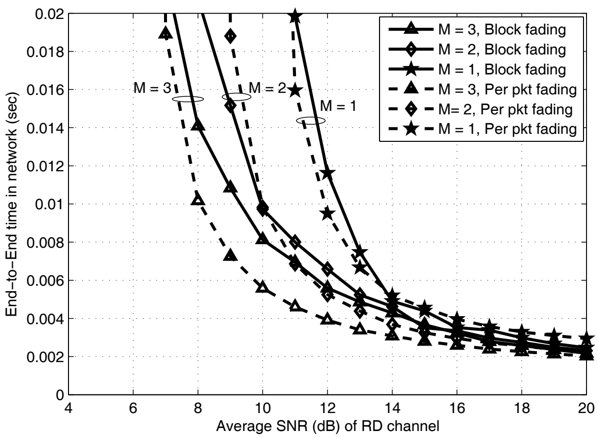

Fig. 9. Comparison of the end-to-end transit time for the block fading channel model and the per packet fading model as a function of the number of relays.

mean ρ_i^{SR}. Therefore,

$$p_{\kappa_i^{\text{SR}}}(x) = \frac{d}{dx} F_{\kappa_i^{\text{SR}}}(x) = \frac{\tilde{B}}{\rho_i^{\text{SR}} x^2} \exp\left(\frac{\tilde{B}}{x} + \frac{1-e^{\tilde{B}/x}}{\rho_i^{\text{SR}}}\right). \tag{40}$$

The PDF of S_i^{SR} without time-out is then

$$\begin{aligned} p_{S_i^{\text{SR}}}(x) &= p(\kappa_i^{\text{SR}} = x | \mathcal{R}_i \text{ is selected}), \\ &= \frac{p(\kappa_i^{\text{SR}} = x \text{ and } \kappa_j^{\text{SR}} \geq x, \ \forall \ j \neq i)}{p_i}. \end{aligned} \tag{41}$$

Therefore, we get $p_{S_i^{\text{SR}}}(x) = \frac{p_{\kappa_i^{\text{SR}}}(x)}{p_i} \prod_{\substack{j=1 \\ (j \neq i)}}^{M} (1 - F_{\kappa_j^{\text{SR}}}(x)) = \frac{\tilde{B}}{p_i \rho_i^{\text{SR}} x^2} \exp\left(\frac{\tilde{B}}{x} + \frac{1-e^{\tilde{B}/x}}{\rho_i^{\text{SR}}}\right) \prod_{\substack{j=1 \\ (j \neq i)}}^{M} \left(1 - \exp\left[\frac{1}{\rho_j^{\text{SR}}}\left(1 - e^{\tilde{B}/x}\right)\right]\right)$. Expanding the product term and simplifying gives

$$p_{S_i^{\text{SR}}}(x) = \frac{\tilde{B}}{p_i \rho_i^{\text{SR}} x^2} \sum_{k=1}^{2^{M-1}} (-1)^{|\omega_k|} \exp\left(\frac{\tilde{B}}{x} + \frac{1-e^{\tilde{B}/x}}{a_{ik}}\right), \tag{42}$$

where a_{ik} and ω_k are as defined in the lemma. Since the total probability mass of $p_{S_i^{\text{SR}}}(x)$ must be 1, the expression for p_i follows from integrating (42) and equating it to 1.

In the presence of time-out, $p_{S_i^{\text{SR}}}(x)$ remains the same as in (42) for $x < t_{\text{out}}$, it equals 0 for $x > t_{\text{out}}$, and is an impulse of magnitude $P_{\text{out},i}^{\text{SR}}$ for $x = t_{\text{out}}$. The probability that the source times out given that $\mathcal{R}_i$ is selected is then $P_{\text{out},i}^{\text{SR}} = \int_{t_{\text{out}}}^{\infty} p_{S_i^{\text{SR}}}(x)\, dx$, which simplifies to (5) after substituting (42) in the integral. Since $S_{\text{nd},i}^{\text{SR}} = \left[S_i^{\text{SR}} | S_i^{\text{SR}} < t_{\text{out}}\right]$, the PDF of $S_{\text{nd},i}^{\text{SR}}$ follows from (42) and Baye's rule.

B. Proof of Lemma 2 (Moments of $S_{nd,i}^{SR}$)

Starting from the PDF of $S_{\text{nd},i}^{\text{SR}}$ in (6) and using the variable substitution $y = \tilde{B}/x$, we have

$$\begin{aligned} \mathbf{E}\left[\left(S_{\text{nd},i}^{\text{SR}}\right)^m\right] &= \frac{\tilde{B}^m}{p_i \rho_i^{\text{SR}} \left(1 - P_{\text{out},i}^{\text{SR}}\right)} \sum_{k=1}^{2^{M-1}} (-1)^{|\omega_k|} \\ &\times \int_{\tilde{B}/t_{\text{out}}}^{\infty} \frac{1}{y^m} \exp\left(y + \frac{1-e^y}{a_{ik}}\right) dy, \quad m = 1, 2. \end{aligned} \tag{43}$$

From the definition of $\psi_m(a,u)$ in (10), the desired result in (9) follows.

C. Proof of Lemma 3 (Moments of $N_j^{(i)}[l]$)

Consider Relay i. The probability that it is selected is p_i. In the lth cycle, let $\mathcal{R}_i$ get chosen only in the $(n+1)$th packet. Therefore, the probability, $P\left(N_j^{(i)}[l] = k \,\middle|\, n\right)$, that $\mathcal{R}_j, j \neq i$, is chosen k times among the first n packets in the lth cycle equals

$$P\left(N_j^{(i)}[l] = k \,\middle|\, n\right) = \binom{n}{k} \left(1 - \frac{p_j}{1-p_i}\right)^{n-k} \left(\frac{p_j}{1-p_i}\right)^k. \tag{44}$$

From this, it can be shown that $\mathbf{E}\left[N_j^{(i)}[l] | n\right] = np_j/(1-p_i)$. We also have $P(n) = p_i(1-p_i)^n$, for $n \geq 0$. Therefore, $\mathbf{E}\left[N_j^{(i)}[l]\right] = \frac{p_j}{1-p_i} \sum_{n=0}^{\infty} np_i(1-p_i)^n = p_j/p_i$. Similarly, $\mathbf{E}\left[\left(N_j^{(i)}[l]\right)^2 | n\right] = \frac{np_j}{1-p_i}\left(1 + (n-1)\frac{p_j}{1-p_i}\right)$. Unconditioning over n yields $\mathbf{E}\left[\left(N_j^{(i)}[l]\right)^2\right] = p_j(2p_j + p_i)/p_i^2$. When the cycles are different, *i.e.*, $l \neq m$, $N_{j_1}^{(i)}[l]$ and $N_{j_2}^{(i)}[m]$ are independent of each other, for all $1 \leq j_1, j_2 \leq M$. Hence, $\mathbf{E}\left[N_{j_1}^{(i)}[l] N_{j_2}^{(i)}[m]\right] = \mathbf{E}\left[N_{j_1}^{(i)}[l]\right] \mathbf{E}\left[N_{j_2}^{(i)}[m]\right] = p_{j_1} p_{j_2}/p_i^2$, for $l \neq m$.

The probability that $\mathcal{R}_{j_1}$ is visited s_1 times and $\mathcal{R}_{j_2}$ is visited s_2 times in the lth cycle is

$$\begin{aligned} P\left(N_{j_1}[l] = s_1, N_{j_2}[l] = s_2 \,\middle|\, n\right) &= \binom{n}{s_1, s_2} \left(\frac{p_{j_1}}{1-p_i}\right)^{s_1} \\ \times \left(\frac{p_{j_2}}{1-p_i}\right)^{s_2} &\left(1 - \frac{p_{j_1} + p_{j_2}}{1-p_i}\right)^{n-s_1-s_2}, \end{aligned} \tag{45}$$

where $\binom{n}{s_1, s_2}$ is the multinomial combinatorial function. From this, it can be shown, after unconditioning on n, that $\mathbf{E}\left[N_{j_1}[l] N_{j_2}[l]\right] = 2p_{j_1}p_{j_2}/p_i^2$, for $j_1 \neq j_2$.

D. Statistics of Inter-Arrival Time for Relay Selection

From (13), $A_i = S_{\text{nd},i}^{\text{SR}} + \beta_i - t_{\text{out}}$, where

$$\beta_i = \sum_{l=1}^{c_i} \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{k=1}^{N_j^{(i)}[l]} S_j^{\text{SR}}[k] + c_i t_{\text{out}}. \tag{46}$$

Note that c_i is independent of S_j^{SR} and $N_j^{(i)}[l]$. Therefore, $\mathbf{E}\left[\beta_i\right] = \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \mathbf{E}\left[N_j^{(i)}\right] \mathbf{E}\left[S_j^{\text{SR}}\right] \mathbf{E}\left[c_i\right] + \mathbf{E}\left[c_i\right] t_{\text{out}}$. And, $P(c_i = l) = \left(P_{\text{out},i}^{\text{SR}}\right)^{l-1} (1 - P_{\text{out},i}^{\text{SR}})$ implies that

$$\mathbf{E}\left[c_i\right] = \frac{1}{1 - P_{\text{out},i}^{\text{SR}}}. \tag{47}$$

Hence, using Lemma 3, we get

$$\mathbf{E}\left[\beta_i\right] = \frac{1}{1 - P_{\text{out},i}^{\text{SR}}} \left(t_{\text{out}} + \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \frac{p_j}{p_i} \mathbf{E}\left[S_j^{\text{SR}}\right]\right). \tag{48}$$

The desired expression for $\mathbf{E}[A_i]$ in (16) then easily follows from (48) and (11).

We have $\mathrm{Var}[A_i] = \mathrm{Var}[\beta_i] + \mathrm{Var}\left[S_{\mathrm{nd},i}^{\mathrm{SR}}\right]$ since $S_{\mathrm{nd},i}^{\mathrm{SR}}$ and β_i are independent. We now evaluate $\mathrm{Var}[\beta_i] = \mathbf{E}\left[\beta_i^2\right] - \mathbf{E}^2[\beta_i]$. The key idea in the derivation below is to cluster the summation terms into those from the same cycle, in which $N_{j_1}^{(i)}$ and $N_{j_2}^{(i)}$ are correlated (see Lemma 3), and into those from different cycles, in which they are uncorrelated. Furthermore, we use the fact that $S_{j_1}^{\mathrm{SR}}[k_1]$ and $S_{j_2}^{\mathrm{SR}}[k_2]$ are independent when *either* $k_1 \neq k_2$ or $j_1 \neq j_2$. Upon carefully expanding the expression for β_i^2, we get (from (46))

$$\begin{aligned}\beta_i^2 &= \sum_{l=1}^{c_i} \sum_{\substack{j=1\\(j\neq i)}}^{M} \sum_{k=1}^{N_j^{(i)}[l]} \left(S_j^{\mathrm{SR}}[k]\right)^2 \\ &\quad + \sum_{l=1}^{c_i} \sum_{\substack{j=1\\(j\neq i)}}^{M} \sum_{k_1=1}^{N_j^{(i)}[l]} \sum_{\substack{k_2=1\\(k_2\neq k_1)}}^{N_j^{(i)}[l]} S_j^{\mathrm{SR}}[k_1] S_j^{\mathrm{SR}}[k_2] \\ &\quad + \sum_{l=1}^{c_i} \sum_{\substack{j_1=1\\(j_1\neq i)}}^{M} \sum_{\substack{j_2=1\\(j_2\neq i,j_1)}}^{M} \sum_{k_1=1}^{N_{j_1}^{(i)}[l]} \sum_{k_2=1}^{N_{j_2}^{(i)}[l]} S_{j_1}^{\mathrm{SR}}[k_1] S_{j_2}^{\mathrm{SR}}[k_2] \\ &\quad + \sum_{l_1=1}^{c_i} \sum_{\substack{l_2=1\\(l_2\neq l_1)}}^{c_i} \left[\sum_{\substack{j_1=1\\(j_1\neq i)}}^{M} \sum_{k=1}^{N_{j_1}^{(i)}[l]} S_{j_1}^{\mathrm{SR}}[k]\right] \left[\sum_{\substack{j_2=1\\(j_2\neq i)}}^{M} \sum_{k=1}^{N_{j_2}^{(i)}[l]} S_{j_2}^{\mathrm{SR}}[k]\right] \\ &\quad + c_i^2 t_{\mathrm{out}}^2 + 2t_{\mathrm{out}} c_i \sum_{l=1}^{c_i} \sum_{\substack{j=1\\(j\neq i)}}^{M} \sum_{k=1}^{N_j^{(i)}[l]} S_j^{\mathrm{SR}}[k]. \quad (49)\end{aligned}$$

Taking expectation of β_i^2, conditioned on c_i, and using the various independence relations, we get

$$\begin{aligned}\mathbf{E}\left[\beta_i^2 | c_i\right] &= c_i \sum_{\substack{j=1\\(j\neq i)}}^{M} \mathbf{E}\left[N_j^{(i)}[l]\right] \mathbf{E}\left[\left(S_j^{\mathrm{SR}}\right)^2\right] \\ &\quad + c_i \sum_{\substack{j=1\\(j\neq i)}}^{M} \mathbf{E}\left[N_j^{(i)}[l]\left(N_j^{(i)}[l]-1\right)\right] \mathbf{E}^2\left[S_j^{\mathrm{SR}}\right] \\ &\quad + c_i \sum_{\substack{j_1=1\\(j_1\neq i)}}^{M} \sum_{\substack{j_2=1\\(j_2\neq i,j_1)}}^{M} \mathbf{E}\left[N_{j_1}^{(i)}[l] N_{j_2}^{(i)}[l]\right] \mathbf{E}\left[S_{j_1}^{\mathrm{SR}}\right] \mathbf{E}\left[S_{j_2}^{\mathrm{SR}}\right] \\ &\quad + c_i (c_i - 1) \left[\sum_{\substack{j_1=1\\(j_1\neq i)}}^{M} \mathbf{E}\left[N_{j_1}^{(i)}\right] \mathbf{E}\left[S_{j_1}^{\mathrm{SR}}\right]\right]^2 \\ &\quad + c_i^2 t_{\mathrm{out}}^2 + 2t_{\mathrm{out}} c_i^2 \sum_{\substack{j=1\\(j\neq i)}}^{M} \mathbf{E}\left[N_j^{(i)}\right] \mathbf{E}\left[S_j^{\mathrm{SR}}\right]. \quad (50)\end{aligned}$$

From Lemma 3 and the probability of c_i given in (14) we get

$$\begin{aligned}\mathbf{E}\left[\beta_i^2\right] &= \frac{1}{1-P_{\mathrm{out},i}^{\mathrm{SR}}} \sum_{\substack{j=1\\(j\neq i)}}^{M} \frac{p_j}{p_i} \mathbf{E}\left[\left(S_j^{\mathrm{SR}}\right)^2\right] \\ &\quad + \frac{2}{1-P_{\mathrm{out},i}^{\mathrm{SR}}} \sum_{\substack{j=1\\(j\neq i)}}^{M} \frac{p_j^2}{p_i^2} \mathbf{E}^2\left[S_j^{\mathrm{SR}}\right] \\ &\quad + \frac{2}{1-P_{\mathrm{out},i}^{\mathrm{SR}}} \sum_{\substack{j_1=1\\(j_1\neq i)}}^{M} \sum_{\substack{j_2=1\\(j_2\neq i,j_1)}}^{M} \frac{p_{j_1} p_{j_2}}{p_i^2} \mathbf{E}\left[S_{j_1}^{\mathrm{SR}}\right] \mathbf{E}\left[S_{j_2}^{\mathrm{SR}}\right] \\ &\quad + \frac{2P_{\mathrm{out},i}^{\mathrm{SR}}}{\left(1-P_{\mathrm{out},i}^{\mathrm{SR}}\right)^2} \left(\sum_{\substack{j=1\\(j\neq i)}}^{M} \frac{p_j}{p_i} \mathbf{E}\left[S_j^{\mathrm{SR}}\right]^2\right)^2 \\ &\quad + t_{\mathrm{out}}^2 \frac{\left(1+P_{\mathrm{out},i}^{\mathrm{SR}}\right)}{\left(1-P_{\mathrm{out},i}^{\mathrm{SR}}\right)^2} + 2t_{\mathrm{out}} \frac{\left(1+P_{\mathrm{out},i}^{\mathrm{SR}}\right)}{\left(1-P_{\mathrm{out},i}^{\mathrm{SR}}\right)^2} \sum_{\substack{j_1=1\\(j_1\neq i)}}^{M} \frac{p_j}{p_i} \mathbf{E}\left[S_j^{\mathrm{SR}}\right]. \quad (51)\end{aligned}$$

Subtracting the expression for $\mathbf{E}^2[\beta_i]$, derived in (48), from (51) and adding $\mathrm{Var}\left[S_{\mathrm{nd},i}^{\mathrm{SR}}\right]$ yields the desired expression for $\mathrm{Var}[A_i]$ in (17).

E. Proof of Theorem 2 (Throughput)

When the queue $\mathcal{Q}_i$ is stable, $\mathbf{E}\left[S_i^{\mathrm{RD}}\right] < \mathbf{E}[A_i]$. In this case, the average rate at which packets are transmitted by $\mathcal{R}_i$ equals the average rate at which packets arrive at $\mathcal{R}_i$, which equals $1/\mathbf{E}[A_i] = 1/\max\left(\mathbf{E}[A_i], \mathbf{E}\left[S_i^{\mathrm{RD}}\right]\right)$. Instead, when $\mathcal{Q}_i$ is unstable, we have $\mathbf{E}\left[S_i^{\mathrm{RD}}\right] \geq \mathbf{E}[A_i]$. The average rate at which packets are transmitted by $\mathcal{R}_i$ then equals $1/\mathbf{E}\left[S_i^{\mathrm{RD}}\right] = 1/\max\left(\mathbf{E}[A_i], \mathbf{E}\left[S_i^{\mathrm{RD}}\right]\right)$ because the $\mathcal{R}_i$–$\mathcal{D}$ link becomes the bottleneck. In both these cases, a fraction $P_{\mathrm{out},i}^{\mathrm{RD}}$ of the packets is lost due to the relay timing out. Therefore, the destination receives packets at an average rate of $\left(1-P_{\mathrm{out},i}^{\mathrm{RD}}\right)/\max\left(\mathbf{E}[A_i], \mathbf{E}\left[S_i^{\mathrm{RD}}\right]\right)$ from $\mathcal{R}_i$. The desired expression follows by summing the throughput from all the relays.

F. Proof of Theorem 3 (E-E Transit Time)

Since the packet is successfully received by $\mathcal{D}$, we know that its transmit time in the network is $S_{\mathrm{nd},i}^{\mathrm{SR}} + W_i + S_{\mathrm{nd},i}^{\mathrm{RD}}$ if it passes through $\mathcal{R}_i$, which is the relay selected for the packet. We, therefore, have

$$\begin{aligned}\xi &= \sum_{i=1}^{M} \left(\mathbf{E}\left[S_{\mathrm{nd},i}^{\mathrm{SR}}\right] + \mathbf{E}[W_i] + \mathbf{E}\left[S_{\mathrm{nd},i}^{\mathrm{RD}}\right]\right) \\ &\quad \times P\left(\mathcal{R}_i \text{ is selected} \,\middle|\, \text{packet received by } \mathcal{D}\right). \quad (52)\end{aligned}$$

The desired result follows from Baye's rule and the following conditional probability relationship:

$$\begin{aligned}&P(\text{packet received by } \mathcal{D} | \mathcal{R}_i \text{ is selected}) \\ &\quad = \left(1-P_{\mathrm{out},i}^{\mathrm{SR}}\right)\left(1-P_{\mathrm{out},i}^{\mathrm{RD}}\right). \quad (53)\end{aligned}$$

G. Proof of Lemma 7 (Moments of Inter-Arrival Time for RR)

As in RS, $A_i = S_{\mathrm{nd},i}^{\mathrm{SR}} - t_{\mathrm{out}} + \beta_i$, where

$$\beta_i = \sum_{l=1}^{c_i} \sum_{\substack{j=1\\(j\neq i)}}^{M} S_j^{\mathrm{SR}}[l] + c_i t_{\mathrm{out}}. \quad (54)$$

Therefore, $\mathbf{E}[\beta_i] = \mathbf{E}[c_i] \sum_{j=1, (j\neq i)}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right] + \mathbf{E}[c_i]\, t_{\text{out}}$. Substituting $\mathbf{E}[c_i] = 1/\left(1 - P_{\text{out},i}^{\text{SR}}\right)$, we get

$$\mathbf{E}[\beta_i] = \frac{1}{1 - P_{\text{out},i}^{\text{SR}}} \left(\sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right] + t_{\text{out}} \right). \quad (55)$$

The desired result for $\mathbf{E}[A_i] = \mathbf{E}\left[S_{\text{nd},i}^{\text{SR}}\right] + \mathbf{E}[\beta_i]$ in (33) then follows easily from (9), (11), and (55).

Also, since β_i is independent of $S_{\text{nd},i}^{\text{SR}}$ for RR, we have

$$\text{Var}[A_i] = \text{Var}\left[S_{\text{nd},i}^{\text{SR}}\right] + \text{Var}[\beta_i]. \quad (56)$$

We now compute $\text{Var}[\beta_i] = \mathbf{E}\left[\beta_i^2\right] - \mathbf{E}^2[\beta_i]$. Proceeding along the same lines as for RS in Appendix D, we get

$$\begin{aligned} \mathbf{E}\left[\beta_i^2 | c_i\right] &= \sum_{l=1}^{c_i} \sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[\left(S_j^{\text{SR}}[l]\right)^2\right] \\ &+ \sum_{l=1}^{c_i} \sum_{\substack{j_1=1 \\ (j_1\neq i)}}^{M} \sum_{\substack{j_2=1 \\ (j_2\neq j_1, i)}}^{M} \mathbf{E}\left[S_{j_1}^{\text{SR}}[l]\right] \mathbf{E}\left[S_{j_2}^{\text{SR}}[l]\right] \\ &+ \sum_{l_1=1}^{c_i} \sum_{\substack{l_2=1 \\ (l_2\neq l_1)}}^{c_i} \left(\sum_{\substack{j_1=1 \\ (j_1\neq i)}}^{M} \mathbf{E}\left[S_{j_1}^{\text{SR}}[l_1]\right] \right) \left(\sum_{\substack{j_2=1 \\ (j_2\neq i)}}^{M} \mathbf{E}\left[S_{j_2}^{\text{SR}}[l_2]\right] \right) \\ &+ c_i^2 t_{\text{out}}^2 + 2 c_i t_{\text{out}} \sum_{l=1}^{c_i} \sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}[l]\right]. \end{aligned} \quad (57)$$

This simplifies to

$$\begin{aligned} \mathbf{E}\left[\beta_i^2 | c_i\right] &= c_i \sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[\left(S_j^{\text{SR}}\right)^2\right] \\ &+ c_i \sum_{\substack{j_1=1 \\ (j_1\neq i)}}^{M} \sum_{\substack{j_2=1 \\ (j_2\neq j_1, i)}}^{M} \mathbf{E}\left[S_{j_1}^{\text{SR}}\right] \mathbf{E}\left[S_{j_2}^{\text{SR}}\right] \\ &+ c_i(c_i - 1) \left(\sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right] \right)^2 \\ &+ c_i^2 t_{\text{out}}^2 + 2 c_i^2 t_{\text{out}} \sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right]. \end{aligned} \quad (58)$$

After averaging over c_i, we get

$$\begin{aligned} \mathbf{E}\left[\beta_i^2\right] &= \frac{1}{1 - P_{\text{out},i}^{\text{SR}}} \sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[\left(S_j^{\text{SR}}\right)^2\right] \\ &+ \frac{1}{1 - P_{\text{out},i}^{\text{SR}}} \sum_{\substack{j_1=1 \\ (j_1\neq i)}}^{M} \sum_{\substack{j_2=1 \\ (j_2\neq j_1, i)}}^{M} \mathbf{E}\left[S_{j_1}^{\text{SR}}\right] \mathbf{E}\left[S_{j_2}^{\text{SR}}\right] \\ &+ \frac{2 P_{\text{out},i}^{\text{SR}}}{\left(1 - P_{\text{out},i}^{\text{SR}}\right)^2} \left(\sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right] \right)^2 + \frac{1 + P_{\text{out},i}^{\text{SR}}}{\left(1 - P_{\text{out},i}^{\text{SR}}\right)^2} t_{\text{out}}^2 \\ &+ \frac{2\left(1 + P_{\text{out},i}^{\text{SR}}\right)}{\left(1 - P_{\text{out},i}^{\text{SR}}\right)^2} t_{\text{out}} \sum_{\substack{j=1 \\ (j\neq i)}}^{M} \mathbf{E}\left[S_j^{\text{SR}}\right]. \end{aligned} \quad (59)$$

Subtracting $\mathbf{E}^2[\beta_i]$, which was derived in (55), from (59) and adding $\text{Var}\left[S_{\text{nd},i}^{\text{SR}}\right]$ results in the desired expression for $\text{Var}[A_i]$ in (34).

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