1. Consider a queue where the customers are arriving according to a Poisson process with rate \( \lambda \).
Compute the aggregate expected waiting time for all arriving customers by time \( t \).

2. Let \( \{Z(t) : t \geq 0\} \) be a compound Poisson process whose jumps occur at rate \( \lambda \) and the iid jump sizes \( \{X_i : i \in \mathbb{N}\} \) are of discrete sizes, taking values in a countable set \( E \subset \mathbb{R} \). Compute the mean function \( E[Z(t)] \) and the moment generating function \( E e^{-\theta Z(t)} \). Compute the moment generating function when jump sizes are not necessarily discrete valued.

3. Consider a Poisson arrival with rate \( \lambda \), where each arrival can be tagged to be of type \( i \in [k] \). At each time \( s \), an arrival can be classified to be of type \( i \) with probability \( p_i(s) \) such that \( \sum_i p_i(s) = 1 \). Let \( N_i(t) \) denote the number of arrivals of type \( i \) in time interval \([0, t]\). Show that \( \{N_i(t) : i \in [k]\} \) are independent with Poisson distribution of mean \( \lambda \int_0^t p_i(s) \, ds \) for type \( i \).

4. Let \( S_n \) be the \( n \)th jump instant of a renewal process with iid inter-renewal time \( X_n \) and renewal function \( m \).
   (a) Compute \( \sum_{n \in \mathbb{N}} P\{S_n \leq t\} \).
   (b) Compute \( E e^{-\theta S_n} \).
   (c) Compute \( E \sum_{n \in \mathbb{N}} f(S_n) \) for any non-negative function on \( \mathbb{R}_+ \).