1. Let \( \{X_t, t \geq 0\} \) be a stochastic process such that
\[
X_t = A \cos(2\pi t + \Theta), \quad t \geq 0.
\]
If \( A \) and \( \Theta \) are independent random variables, and \( \Theta \) is uniformly distributed between \((-\pi, \pi]\).
(a) Compute the following functions \( m_X(t) \), \( R_X(t, s) \), and \( C_X(t, s) \).
(b) Compute the finite dimensional distribution of \( X_t \) when \( A \) is uniform between \([0, 1]\).
(c) Is the process stationary? In what sense?

2. Let \( \{X_n, n \in \mathbb{N}\} \) be a Bernoulli process with success probability \( p \) in a single trial. Let \( \{N_n, n \in \mathbb{N}\} \) be the number of successes in first \( n \) trials.
(a) Compute the following probability \( P\{N_5 = 4, N_7 = 5, N_{13} = 8\} \).
(b) Compute the expectation \( E(N_5 N_8) \).

3. Consider a simple point process \( \{N_t, t \geq 0\} \) with independent and stationary increments.
(a) Show that \( \{N_t = 0\} = e^{-\lambda t} \) for some constant \( \lambda \geq 0 \).
(b) Let \( G(t) = \mathbb{E}e^{\alpha N_t} \), then show that \( G(t) = e^{-\lambda t(1-\alpha)} \).
(c) Show that \( P\{N_t = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \).
(d) Compute \( \text{Var}N_t \).

4. Let \( S_n \) be the \( n \)th jump instant of a Poisson process.
(a) Compute \( \sum_{n \in \mathbb{N}} P\{S_n \leq t\} \).
(b) Compute \( \mathbb{E}e^{-\theta S_n} \).
(c) Compute \( \mathbb{E}\sum_{n \in \mathbb{N}} f(S_n) \) for any non-negative function on \( \mathbb{R}_+ \).