1. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with the common distribution function $F$ defined as

$$F(x) = \Pr\{X_i \leq x\} = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0. \end{cases}$$

Find the mean of the following random variable, defined as

$$Z \triangleq \min\{X_1, X_2, \ldots, X_n\}.$$

Solution: It is easy to see that the following set equality holds true,

$$\{\min\{X_1, \ldots, X_n\} > z\} = \bigcap_{i=1}^{n}\{X_i > z\}.$$

Since $X_1, \ldots, X_n$ are iid random variables, taking the probability measure of events on both sides, we obtain

$$P\{Z \leq z\} = 1 - (1 - F(z))^n = \begin{cases} 1 - e^{-n\lambda z}, & z > 0, \\ 0, & z \leq 0. \end{cases}$$

The mean of an exponential random variable is reciprocal of its rate. Hence, $\mathbb{E}Z = \frac{1}{n\lambda}$.