1. If $X$ and $Y$ are independent gamma random variables with parameters $(\alpha, \lambda)$ and $(\beta, \lambda)$, respectively, compute the joint density of $U = X + Y$ and $V = X / (X + Y)$.

Note, density of a gamma random variable $X$ with parameters $(\alpha, \lambda)$ is defined as follows:

$$f_X(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

where, $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$.

2. Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500. What can be said about the probability that this weeks production will be at least 1000?

3. Suppose that the expected number of accidents per week at an industrial plant is four. Suppose also that the numbers of workers injured in each accident are independent random variables with a common mean of 2. Assume also that the number of workers injured in each accident is independent of the number of accidents that occur. What is the expected number of injuries during a week?

4. Miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door leads to a tunnel that returns him to his mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until the miner reaches safety?

5. Let $X_1 \sim \exp(\lambda_1)$, $X_2 \sim \exp(\lambda_2)$ be independent. Let $R = \frac{X_1}{X_2}$. Compute the pdf of $R$.

6. Suppose $X$ and $Y$ are random variables such that $E[X^2] < \infty$ and $E[Y^2] < \infty$. Further, let $E[X|Y] = Y$ and $E[Y|X] = X$. Then, show that $X = Y$ almost surely.

7. Let $X_1, X_2$ be independent zero mean Gaussians with variance $\sigma^2$. Define

$$R = \sqrt{X_1^2 + X_2^2}, \quad \Theta = \tan^{-1}\left(\frac{X_2}{X_1}\right).$$

1. Compute the densities of $R$ and $\Theta$.
2. Are $R$ and $\Theta$ independent?
3. Let $S = X_1^2 + X_2^2 = R^2$. What is the distribution of $S$?