1. Suppose $N(t)$ is a homogeneous Poisson process with rate $\lambda > 0$. Let $B(t)$ be a continuous time Bernoulli($p$) process independent of $N(t)$, i.e., for all $k \geq 1$ and for all $i_1, i_2, \ldots, i_k$,

$$P(B(t_1) = i_1, B(t_2) = i_2, \ldots, B(t_k) = i_k) = p^{|i_1|} \prod_{j=1}^{k} (1 - p)^{|i_j|}$$

for all $i_1, i_2, \ldots, i_k \in \{0, 1\}$. Let $M(t) = N(t) + B(t)$. Is $M(t)$ a Poisson process?

2. Let $\{X(t) : t \geq 0\}$ be a compound Poisson process with $X(t) = \sum_{i=1}^{N(t)} X_i$, where $N(t)$ is a homogeneous Poisson process with rate $\lambda = 1$ and $X_i$'s are iid with $P(X_i = j) = \frac{j}{10}$ for $j = 1, 2, 3, 4$.

   (a) Compute $P(X(4) = 20)$.
   (b) Evaluate the mean and variance of $X(t)$.

3. Let $N(t)$ be a non-homogeneous Poisson process with rate function $\lambda(t), t \geq 0$. Let $S_1$ and $S_2$ denote the first and second arrival times, and let $X_1 = S_1$ and $X_2 = S_2 - S_1$ be the corresponding interarrival times. Derive an expression for the probability $P(X_2 > t | X_1 = s)$ for $s, t \geq 0$, in terms of the rate function $\lambda(t)$. Express your answer in terms of the function $m(t) = \int_0^t \lambda(y) \, dy$.

4. A shop is open from 10am to 6pm. Customers arrive at the shop according to a Poisson process with an intensity function, which equals zero at opening, 4 customers per hour at noon, 6 customers per hour at 2pm, 2 customers per hour at 4pm and zero at closing. In between any two consecutive times, the arrival rate is linear.

   (a) Find the distribution of the number of customers on a given day.
   (b) Find the probability that no customer arrives until noon.
   (c) Assuming that during the first two hours after opening, exactly two customers have arrived, find the their expected arrival times.

5. Suppose that $N_1(t)$ and $N_2(t)$ are two independent homogeneous Poisson processes with rates $\lambda_1$ and $\lambda_2$ respectively. Upon merging the two processes, what is the probability that for the first $n$ outcomes of the merged process, exactly $k$ of them are from $N_1(t)$ process?

6. Let $N_1(t)$ and $N_2(t)$ be two independent homogeneous Poisson processes with rates $\lambda_1 = 1$ and $\lambda_2 = 2$ respectively. Find the probability that the third arrival in $N_1(t)$ occurs before the third arrival in $N_2(t)$.

7. A married couple is searching for a used car. Each one of them is looking up offers for their favorite brand. The offers for wife’s brand arrive according to a homogeneous Poisson process with intensity $\lambda$, while the offers for husband’s brand arrive according to a homogeneous Poisson process with intensity $\mu$. The wife is ready to buy a car when she encounters the third offer for her brand, while the husband is ready to buy when he encounters the second offer for his brand. The couple buys a car when either of them is ready to buy. We assume that the offer processes for both brands are independent of one another.

   (a) Compute the probability that the couple buys a car according to the wife’s choice.
   (b) Compute the expected time needed to buy a car.

8. Suppose that the number of customers visiting Prakruti in a given time interval $I$ is $N \sim \text{Poisson}(\mu)$. Assume that each customer purchases coffee with probability $p$ independent of other customers and independent of $N$. Let $X$ be the number of customers who purchase coffee in that time interval and $Y$ be the number of those who do not. Thus, $N = X + Y$.

   1. Calculate the marginal distributions of $X$ and $Y$.
   2. Show that $X$ and $Y$ are independent.