1. Let a random variable $X$ be uniform over the interval $[a, b]$. Show that the variance of $X$ is $\frac{(b-a)^2}{12}$.

2. Prove or give a counterexample: for random variables $X, Y, Z$ and real numbers $a, b$, we always have
   \[ \text{Cov}(aX + bY, Z) = a \cdot \text{Cov}(X, Z) + b \cdot \text{Cov}(Y, Z). \]

3. Consider two independent geometric random variables $X_1$ and $X_2$ with parameters $p_1$ and $p_2$ respectively. Define a new random variable $Z$ as $Z := \min(X_1, X_2)$.
   (a) Prove that $Z$ is also a geometric random variable. What is its parameter?
   (b) Compute the expectation of $Z$.

4. Let $(\Omega, \mathcal{F}, P)$ be a probability space, and suppose that $X$ and $Y$ are two $\mathcal{F}$-measurable random variables such that $E[X], E[Y]$ and $E[XY]$ all exist.
   (a) Prove that if $X$ and $Y$ are independent, then $E[XY] = E[X] \cdot E[Y]$.
   (b) Is the converse true? Prove or give a counterexample.

5. Let $X$ be a nonnegative random variable with CDF $F_X$. Then, show that
   \[ E[X] = \int_0^\infty (1 - F_X(x)) \, dx. \]
   (Hint: Prove this for discrete random variables by using “area under the curve interpretation” for an integral. For continuous random variables, you may assume that $F_X(x)$ is differentiable, with \( \frac{d}{dx} (1 - F_X(x)) = -f_X(x) \), and use integration by parts on the formula $E[X] = \int_0^\infty x f_X(x) \, dx$.)

6. Let $Z$ be an exponential random variable with parameter $\lambda > 0$. Show that for all $t_1 > 0$ and $t_2 > 0$,
   \[ P(Z > t_1 + t_2 | Z > t_1) = P(Z > t_2). \]
   This property is called the memoryless property of exponential distribution.

7. Consider a probability space $(\Omega, \mathcal{F}, P)$. Let $X$ and $Y$ be two discrete $\mathcal{F}$-measurable random variables. Further, let $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be two functions such that $g(X)$ and $h(Y)$ are both $\mathcal{F}$-measurable random variables.
   Show that if $X$ and $Y$ are independent, then so are $g(X)$ and $h(Y)$.

8. Virat and Anushka have a date at 7 pm, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. Find the probability that both will meet.