LDPC Codes

1. In the density evolution analysis of \((d_v, d_c)\)-regular LDPC codes, where the goal is to determine the evolution of the density of number of incorrect messages passed between variable nodes and check nodes, it is customary to assume that the all-1 codeword is transmitted. What are the assumptions on the channel and the processing carried out at the variable and check node under which this assumption is valid? Explain your answer while making clear any notation that you introduce.

2. Consider density evolution associated to Gallager Decoding Algorithm A applied to an LDPC code \(C\). Thus the channel is a BSC with cross-over probability \(\epsilon << 1\) and all messages passed are either 1 or \(-1\). You may assume that the neighborhood of every node in the Tanner graph of \(C\) is tree-like to depth 8. What is the probability \(p_{-1}^{(1)}\) that at the end of iteration 1, the message passed from a variable node to check node will be in error. Notation is as in class. Following an initial round of message passing, from the variable nodes to check nodes, based only on channel inputs, each subsequent iteration is composed of two rounds of message passing: from check node to variable node followed by from variable node back to check node. Show all your working clearly. You may use the fact that \(\epsilon << 1\) to simplify calculations. Hence \(a\epsilon^2\) for integer constants \(a < 100\) (say) may safely be ignored in comparison with \(\epsilon\), etc.

3. Consider the variation of belief propagation decoding of binary LDPC codes in which, in place of beliefs, the messages passed correspond to log-likelihood ratios (as discussed in class).

   (a) Identify (it is not necessary to derive them) the variable and check node maps

   \[\psi_v^{(0)}(l_0), \quad \psi_v^{(l)}(l_0, l_1, l_2, \ldots, l_{d_v-1}), \quad \psi_c^{(l)}(l_1, l_2, \ldots, l_{d_c-1}).\]

   (b) Do these maps satisfy the variable-node and check-node symmetry conditions which (along with the channel symmetry condition) permit us to conclude that the number of incorrect messages passed is the same regardless of the transmitted codeword? Make clear your reasoning.

4. Is the computational tree associated with variable node 11 (at the top of the graph and incorrectly labelled as node 10 :-) ) in the Tanner graph in Fig. 1 of a certain LDPC code a junction tree? If so, identify the associated MPF problem along with the local domains and the local kernels. What is the objective function being computed if messages are passed as indicated by the arrows?
5. (a) In the context of the performance analysis of LDPC codes, state (in terms of the notation introduced in class), the variable and check-node symmetry assumptions that go into showing that the probability of passing an incorrect message is independent of the transmitted codeword.

(b) Show clearly that the check-node symmetry condition holds when LDPC codes are decoding using belief propagation with log-likelihood ratios (LLR) in place of beliefs.