1. A sequence of message bits is to be transmitted across a memoryless binary-input, binary-output channel with transition probability \( p(y|x) = \text{Prob}[y \text{ received} | x \text{ transmitted}] \) given by

\[
\begin{align*}
p(0|0) &= \frac{3}{4} & p(1|1) &= \frac{2}{3} \\
p(1|0) &= \frac{1}{4} & p(0|1) &= \frac{1}{3}
\end{align*}
\]

The message is encoded using the rate 1:2 convolutional encoder shown in the figure below. At the start of the encoding process, both the memory elements are initialized to 0, so the initial state of the encoder is 00.

![Convolutional Encoder Diagram](image)

The encoded sequence of bits is transmitted across the channel, and the sequence received at the channel output is \( y = 00 \ 01 \ 01 \ 00 \ 10 \ 10 \ 00 \ 11 \).

(a) Suppose that you are given the information that the encoding process terminated at the 00 state. Use the Viterbi algorithm to determine the most likely transmitted message.

(b) Would your estimate of the transmitted message change if the terminal state of the encoding process was not known?

2. The figure below shows a rate 1 : 2 encoder for a convolutional code over the binary field.

![Convolutional Encoder Diagram](image)

(a) Write down the generator matrix \( G(x) \) corresponding to this encoder.

(b) Determine whether or not the encoder above is catastrophic. If it is catastrophic, find an infinite-weight input sequence that generates a codeword of finite weight.

(c) If the encoder shown is catastrophic, derive a non-catastrophic encoder for the same code. If the encoder shown is non-catastrophic, derive a catastrophic encoder for the same code. In either case, you must give a diagrammatic representation (using adders and delay elements) of the encoder you derive.