1. The state and trellis diagrams for the encoder are depicted in the figure below:

For ML decoding, the Viterbi algorithm is implemented using the branch metrics \( \mu(y_t, c_t) = -\log p(y_t|c_t) \).

(a) In this case, we are looking for the path having the least path metric among all paths starting in state 00 at time \( t = 0 \) and ending in state 00 at time \( t = 8 \). The Viterbi algorithm determines this path to be one driven by the input sequence \( u = 0 1 0 1 1 1 0 1 \), and producing the codeword \( c = 00 11 01 10 10 10 01 11 \). Thus, the most likely message sequence is \( u = 0 1 0 1 1 1 0 1 \).

(b) Since the terminal state is not known, we have to find the path having the least path metric among all paths starting in state 00 at time \( t = 0 \) and ending at any of the 4 possible states at time \( t = 8 \). From the Viterbi algorithm, we now find that there are two paths having least path metric. One path is the same as that in part (a). The second path ends at state 10: it is determined by the input sequence \( u = 1 0 0 0 1 1 0 1 \), and produces the codeword \( c = 11 01 01 00 10 11 00 11 \). Thus, the most likely message sequence is either \( u = 0 1 0 1 1 1 0 1 \) or \( u = 1 0 0 0 1 1 0 1 \).

2. (a) \( G(x) = [1 + x + x^3, 1 + x + x^2, 1 + x^2 + x^3] \). The gcd of the three polynomials in \( G(x) \) is 1. Hence, the corresponding encoder is not catastrophic.

(b) \( G(x) = [1 + x^3, 1 + x + x^2 + x^4, 1 + x^2 + x^3 + x^4] \). In this case, the gcd of the three polynomials is \( 1 + x \). Hence, the corresponding encoder is catastrophic. Note that with the infinite-weight input \( u(x) = \sum_{i=0}^{\infty} x^i = \frac{1}{1+x} \), the output codeword is

\[
c(x) = u(x)G(x) = [1 + x + x^2, 1 + x^2 + x^3, 1 + x + x^3],
\]

which is of finite weight.