1. A sequence of message bits is to be transmitted across a memoryless binary-input, binary-output channel with transition probability \( p(y|x) = \text{Prob}[y \text{ received} \mid x \text{ transmitted}] \) given by

\[
\begin{align*}
p(0|0) &= \frac{3}{4} & p(1|1) &= \frac{2}{3} \\
p(1|0) &= \frac{1}{4} & p(0|1) &= \frac{1}{3}
\end{align*}
\]

The message is encoded using the rate 1:2 convolutional encoder shown in the figure below. At the start of the encoding process, both the memory elements are initialized to 0, so the initial state of the encoder is 00.

The encoded sequence of bits is transmitted across the channel, and the sequence received at the channel output is \( y = 00 \ 01 \ 01 \ 00 \ 10 \ 10 \ 00 \ 11 \).

(a) Suppose that you are given the information that the encoding process terminated at the 00 state. Use the Viterbi algorithm to determine the most likely transmitted message.

(b) Would your estimate of the transmitted message change if the terminal state of the encoding process was not known?

2. Determine whether the convolutional codes over \( \mathbb{F}_2 \) encoded using the generator matrices given below are catastrophic. If so, find an infinite-weight input sequence that generates a codeword of finite weight.

(a) \( G(x) = [1 + x + x^3, \ 1 + x + x^2, \ 1 + x^2 + x^3] \)

(b) \( G(x) = [1 + x^3, \ 1 + x + x^2 + x^4, \ 1 + x^2 + x^3 + x^4] \)