E2 205 (Aug–Dec 2018)

Homework Assignment 1

1. The International Standard Book Number (ISBN) system, used to uniquely identify books, is a non-binary code defined over the 11-letter alphabet \{0, 1, 2, \ldots, 9, X\}, where \(X\) represents the number 10. An ISBN codeword, \(c_1c_2 \ldots c_{10}\), is ten digits in length. The first nine digits are “information digits” that record data such as publisher, title and edition. The tenth digit is a check digit, chosen so that

\[
\sum_{i=1}^{10} i \cdot c_i \equiv 0 \pmod{11}
\]

In this application, the common types of error are digits being wrong, and transpositions, in which some two digits, \(c_i\) and \(c_j\), are interchanged. Analyze the error detection/correction capability of the ISBN code against these types of errors.

2. A \(k\)-bit message is to be transmitted across a memoryless binary symmetric channel with probability of error \(p = 0.1\). The error-protection scheme used is an \(r\)-fold repetition code, in which each message bit is repeated \(r\) times before transmission, and a majority rule is used for decoding at the channel output. Given a value of \(k\), let \(r(k)\) denote the minimum value of \(r\) for which the probability that the entire \(k\)-bit message will be received and decoded correctly is at least 0.999.

(a) Determine \(r(1)\), \(r(2)\) and \(r(3)\).

(b) Show that \(r(k)\) is a non-decreasing function of \(k\), i.e., \(k_1 \leq k_2 \Rightarrow r(k_1) \leq r(k_2)\).

(c) Show that \(r(k)\) is unbounded, i.e., there is no real number \(R\) such that \(r(k) \leq R\) for all \(k\).

This problem shows that repetition codes are poor coding schemes. Note that \(\frac{1}{r(k)}\) denotes the coding rate required to ensure a 99.9\% chance of successful transmission of a \(k\)-bit message using a repetition code. Parts (b) and (c) of the problem show that \(\lim_{k \to \infty} r(k) = \infty\). Thus, the coding rate must go to zero as the length of the message increases, to maintain reliable transmission.

3. Let \(C\) be the length-7 Hamming code, as described in class. What is the minimum distance of \(C\)?

4. Let \(C\) be a block code with minimum distance \(d\), and let \(\rho, \tau\) be non-negative integers such that \(2\tau + \rho \leq d - 1\). Specify a decoder for \(C\) that can correct every error-cum-erasure pattern that contains at most \(\tau\) errors along with at most \(\rho\) erasures.