1. Let $\{N(t), t \geq 0\}$ be a Poisson process of rate $\lambda > 0$, and let $T_k = \inf\{t \geq 0 : N(t) \geq k\}$, $k = 1, 2, \ldots$, be the $k$th arrival (or jump) times. Prove that for any $k \geq 1$, we have $\lim_{t \to \infty} P(T_k > t) = 0$, and hence, $P(T_k < \infty) = 1$.

2. Problem 3.9 in Prof. AK's SPQT notes.

3. Let $N(t) = N_1(t) + N_2(t)$ be the superposition of two independent Poisson processes. Prove that $\{N(t)\}$ satisfies the independent increments property.

4. Let $\{N_1(t)\}$ and $\{N_2(t)\}$ be two independent Poisson processes with positive rates $\lambda_1$ and $\lambda_2$, respectively, and let $N(t) = N_1(t) + N_2(t)$ denote their superposition. Now, consider a splitting of $\{N(t)\}$ in which each arrival of $\{N(t)\}$ gets labelled 1 with probability $\frac{\lambda_1}{\lambda_1 + \lambda_2}$, and 2 with probability $\frac{\lambda_2}{\lambda_1 + \lambda_2}$, independently of other arrivals. Let $\{\tilde{N}_i(t)\}$, $i = 1, 2$, denote the counting process formed by the arrivals labelled $i$. By the splitting theorem for Poisson processes, $\{\tilde{N}_1(t)\}$ and $\{\tilde{N}_2(t)\}$ are independent Poisson processes with rates $\lambda_1$ and $\lambda_2$, respectively. Thus, the processes $\{\tilde{N}_1(t)\}$ and $\{\tilde{N}_2(t)\}$ have the same joint distribution as the original processes $\{N_1(t)\}$ and $\{N_2(t)\}$.

Now, work out Problem 3.5 in Prof. AK's SPQT notes. ("epoch" $\equiv$ "arrival time")