Minimum Error Probability MIMO-Aided Relaying: Multi-Hop, Parallel and Cognitive Designs

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Abstract—A design methodology based on the minimum error probability (MEP) framework is proposed for a non-regenerative multiple-input multiple-output (MIMO) relay-aided system. We consider the associated cognitive, the parallel and the multi-hop source-relay-destination (SRD) link design based on this MEP framework, including the transmit precoder, the amplify-and-forward (AF) relay matrix and the receiver equalizer matrix of our system. It has been shown in the literature that MEP based communication systems are capable of improving the error probability of other linear counterparts. Our simulation results demonstrate that the proposed scheme indeed achieves a significant BER reduction over the existing linear schemes.

Index Terms—LMMSE, MEP, MC, MIMO, Relay, Cognitive.

I. INTRODUCTION

MIMO relaying is becoming an eminent and integral part of advanced wireless communication systems [1], owing to its capability of enhancing the received signal. The joint design of the transmitter of the relay and of the destination receiver along with the MIMO benefits has attracted tremendous research namely multi-hop relays, parallel relays and a relay-aided cognitive, have been considered by numerous researchers for tackling a range of challenges, including the coverage range extension [2], [3] and the careful choice of the best links from the entire set of legitimate links [4].

Numerous design criteria, such as the mean square error (MSE), the maximization of the capacity (MC) and various others, have been used for MIMO-aided relaying in the literature. For example, multi-hop relaying, which is capable of substantially extending the cellular coverage, has been designed relying on the MSE criterion [2], [3]. On the other hand, the so-called parallel relay configuration [4], which allows the best relay link to be selected from a set of parallel relay links used the MSE criterion for designing the relaying weights. Cognitive communications, where the bandwidth is judiciously shared between the primary and secondary users, has also been extended to the family of MIMO relay-aided systems [5], [6] using the MC criterion. However, a fundamental limitation of these criteria is that they are unable to achieve the minimum-error-probability (MEP), i.e. the lowest bit-error-ratio (BER) in a linear detection framework [7]. Hence, the MEP based transceiver design criterion, also known as the minimum BER (MBER) method, is a more pertinent design criterion as far as the BER performance is concerned. Although, the benefits of the MEP-based MIMO-relaying system have already been demonstrated in [8] in terms of an SNR gain of up to 3 – 4 dB, in this treatise our holistic CF is conceived in the above mentioned scenarios equipped with MIMO configurations for the first time.

Against this background, we propose to invoke the MEP optimization criterion as our objective function for jointly optimizing the transmit precoder (TPC), the amplify-and-forward (AF) MIMO-weights for the relays and the equalizer weights for the destination of three different relaying topologies - namely the multi-hop, the parallel and the cognitive relaying regimes. We develop the MEP based cost function (CF) for these three network topologies based on the classic QPSK and QAM signal constellation. We opted for the projected steepest descent (PSD) [9] optimization tool for finding the minimum of the CF. Our numerical simulations demonstrate that this criterion leads to significantly lower BER than its counterparts.

II. SYSTEM MODEL

A. Cognitive MIMO-relay model

For the cognitive MIMO-relay, we consider a single-hop relaying system consisting of a source node (SN), a relay node (RN) and a destination node (DN) having $N_s$, $N_r$, and $N_d$ antennas, respectively, as shown in Fig. 1. Let us assume that the primary user (PU), sharing the same bandwidth and having $N_p$ receiver antenna suffers from interference from RN [5]. Let us denote that $N_s$ is the length of the input vector $x \in \mathbb{C}^{N_s \times 1}$ before the TPC operation at the SN, where $A_s \in \mathbb{C}^{N_s \times N_s}$ is the TPC matrix. We denote $H_{sr} \in \mathbb{C}^{N_r \times N_s}$, $H_{rd} \in \mathbb{C}^{N_d \times N_r}$ and $H_{dp} \in \mathbb{C}^{N_p \times N_d}$ as the SN-RN, RN-DN and SN-PU channel gain matrices, respectively. Let us denote the i.i.d AWGN vectors at the RN and DN as $v_r \in \mathbb{C}^{N_r \times 1}$ and $v_d \in \mathbb{C}^{N_d \times 1}$, with the variance of $\sigma^2_{v_r}$ and $\sigma^2_{v_d}$ for each component, respectively. Thus, the vector received at the RN is given by

$$r_r = H_{sr}A_s x + v_r, \quad (1)$$

Let us denote the AF matrix by $A_F \in \mathbb{C}^{N_r \times N_s}$. The power constraint at the RN is calculated as,

$$T_F \left[ A_F \left( \sigma^2_{v_r} H_{sr} A_s A_s^H H_{rd}^H + \sigma^2_{v_d} I_{N_r} \right) A_F^H \right] \leq P_r, \quad (2)$$

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where $P_r$ is the RN’s transmit power and $\mathbb{E}(xx^H) = \sigma_d^2 I_N$. We also calculate the average interference ($I_p$) at the PU as

$$T_r \left[ H_{r_p} A_j H_j^H H_{r_p}^H + \rho_1 H_{r_p} A_j A_j^H H_{r_p}^H \right] \leq I_p/\sigma_r^2$$

(3)

where, $\rho_1 = I_p/\sigma_r^2$. Similarly, we obtain the received signal at the DN as

$$r_d = H_{rd}(A_r A_S x + A_{F,k} v_r + v_d)$$

$\triangleq Hx + v,$

(4)

where $H \triangleq H_{rd} A_r A_S$ and $v \triangleq H_{rd} A_r v_r + v_d$, while $v_d$ is the noise at DN, which has a covariance matrix of $\sigma_d^2 I_N$. The effective noise $v$ has a covariance matrix of $C_v = \sigma_d^2 I_N + H_{rd} A_r A_r^H H_{rd}^H$. An equalizer matrix $W_d = C_{v_d}^{-1}$ used at the DN would estimate the vector $x$ by $\hat{x} = W_d^H r_d$.

**B. Parallel MIMO-relay model**

For the parallel MIMO-relay, our final design goal is to select the best relay link from the set of $K$ parallel relay links between the SN and the DN, as shown in Fig. 2. Let us assume that $H_{sr}^k, H_{rd}^k$ and $A_{F,k}$ denote the SN-RN, RN-DN channels and the AF matrices (w.r.t kth, RN), respectively. The data received at the $k^{th}$ relay after multiplication by the AF relaying-matrix is given by,

$$r_{rk} = A_{F,k} H_{sr,k} A_S x + A_{F,k} v_r,$$

(5)

with the power constraint formulated as

$$T_r \left[ A_{F,k} (\sigma_d^2 H_{sr,k} A_S A_S^H (H_{sr,k})^H + \sigma_d^2 I_N) \right] \leq P_r$$

(6)

We assume that each link has a maximum power budget of $P_r$. The data received at the DN from the $k^{th}$ relay link is given by,

$$r_{dk} = H_{rd,k} A_{F,k} H_{sr,k} A_S x + H_{rd,k} A_{F,k} v_r + v_d.$$ 

(7)

**C. Multi-hop MIMO-relay model**

For the multi-hop MIMO-relay scenario, we assume that there are $K$ recursive single relays, as shown in Fig. 3. For simplicity, we assume having a single source and a destination node. The matrices $H_{sr,k} \in \mathbb{C}^{N_r \times N_t}$ and $A_{F,k} \in \mathbb{C}^{N_t \times N_t}$ represent the $(k-1)^{th}$ to $k^{th}$ relay link and the AF relaying-matrix of the $k^{th}$ RN, respectively. We impose the power constraint of $P_{sr,k}$ at the $k^{th}$ RN. Hence, the signal received at the $k^{th}$ RN after multiplication by the AF relaying-matrix becomes $[2], [3]

$$r_{dk} = \prod_{i=1}^{k-1} (H_{r_i} A_{F,i}) A_S x + \prod_{j=1}^{k} (H_{r_j} A_{F,j}) v_{r,j-1} + v_{r,k}$$

(8)

Similarly, the signal received at the DN is given by

$$r_d = H_{rd} A_{F,K} \prod_{k=1}^{K-1} (H_{r_i} A_{F,i}) A_S x +$$

$$\prod_{j=1}^{K-1} (H_{r_j} A_{F,j}) v_{r,j-1} + v_{r,K-1} + v_d.$$

$\triangleq Hx + v,$

(9)

where $H$ and $v$ are defined as follows

$$H \triangleq H_{rd} A_{F,K} \prod_{k=1}^{K-1} (H_{r_i} A_{F,i}) A_S,$$

(10)

$$v \triangleq H_{rd} A_{F,K-1} \prod_{j=2}^{K-1} (H_{r_j} A_{F,j}) v_{r,j-1} + v_{r,K-1} + v_d.$$ 

The overall covariance matrix is then defined as

$$C_v = \sum_{k=2}^{K} \sigma_d^2 \left( \prod_{i=k}^{K} H_{r_i} A_{F,i} \right) \left( \prod_{i=k}^{K} H_{r_i} A_{F,i} \right)^H + \sigma_d^2 I_N.$$ 

(11)

We assume that the channel state information (CSI) required at various nodes as depicted in Table I. We assume that DN and the primary user send the CSI to the RN through feedback channel.

<table>
<thead>
<tr>
<th>Link</th>
<th>SN</th>
<th>RN</th>
<th>DN</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN-RN-DN</td>
<td>$H_{sr}$, $H_{rd}$, $H_{rp}$</td>
<td>$H_{rd}$</td>
<td>$H_{rd}$</td>
</tr>
</tbody>
</table>

III. MEP COST FUNCTION (CF)

In the current context, the MEP CF directly minimizes the BER of the system at the DN. We start the formulation of the MEP CF with QPSK constellation and then extend it to the QAM case. Let us denote the symbol error ratio (SER) by $P_{se}$, when detecting $x_i$ (the $i^{th}$ component of $x$) at the DN. With a slight ‘abuse’ of notation, we consider the SER here instead of BER, since the BER and SER are
approximately related to each other as SER \approx \log_2(M) \times \text{BER}

in conjunction with grey coding. If \hat{x}_i is detected independently, the average probability of a symbol error associated with detecting the complete vector x is given by

\[ P_e = \frac{1}{N_s} \sum_{i=1}^{N_s} P_{c,i}. \]  \hfill (12)

Let us denote w_i as the i-th column of the DN’s equalizer matrix W_d. Assume that L = 2^{N_c} represents the total number of unique realizations of x, while x_i is the j-th such realization of x. For the Gaussian Q(x)-function, we use an approximation, which works well for a good range of x. This is given as \[ Q(x) = K_c \exp\left(-\frac{m_c x^2}{2}\right). \]  \hfill (13)

where m_c is chosen from 1 \leq x \leq 2 and K_c is function of m_c as defined in [10]. If \hat{x}_i is the estimate of x_i for the QPSK constellation, we arrive at the expression of \( P_{c,i} \) in (14) [8].

A. CF with M-QAM constellation

Here the CF formulation is extended to the general M-QAM constellation. Let us assume that 2a (for any \( a > 0 \)) denotes the distance between two adjacent constellation points along either the real or imaginary axis. The M-QAM constellation can be interpreted as two orthogonal PAM sequences of length \( \sqrt{M} \). Therefore, the SER can be obtained as,

\[ P_{QAM}^{c,i} = 1 - P_{c,i}^R \cdot P_{c,i}^I, \]  \hfill (15)

where \( P_{c,i}^R, P_{c,i}^I \) represent the probability of correct decision along the real and imaginary axes, respectively. For computational simplicity, we assume that the decision region of each point along either the real or imaginary axis is bounded by 2a, although this can be exceeded with a small probability. Let us define \( L_i = M^{(N_c - 1) \over 2} \). Now, \( P_{c,i}^R, P_{c,i}^I \) are derived in equations (16) and (18), respectively.

B. Optimization problem

We now have to obtain the optimal TPC weights as well as the AF and equalizer matrices by optimizing the CF. Hence, for the cognitive case, the optimization problem can be stated as

\[ \begin{align*}
A_{\text{S}}^{\text{mep}}, A_{\text{F}}^{\text{mep}}, W_d^{\text{mep}} &= \arg\min_{A_{\text{S}}, A_{\text{F}}, W_d} P_e(A_{\text{S}}, A_{\text{F}}, W_d) \\
\text{s.t.} & \quad \text{(1)} \quad \text{Tr}\left[A_F \left(\sigma_0^2 C_r^{-1} H_{sr} A_S A_S^H (C_r^H)^{-1} + I_{N_r}\right) A_F^H\right] \leq P_t, \\
 & \quad \text{(2)} \quad \sigma_0^2 \text{Tr}(A_F^H A_S) \leq P_t, \\
 & \quad \text{(3)} \quad \text{Tr}\left[H_{rp} A_F A_F^H H_{rp}^H + \rho_1 H_{rp} A_F A_S A_F^H A_S^H H_{rp}^H \right] \leq I_p / \sigma_r^2.
\end{align*} \]  \hfill (20)

For the parallel relaying case, this is a two-step process. In the first step, we optimize each parallel link independently as per equation similar to (20) and then during the second step, we choose the specific link having the lowest value of the CF, i.e. the lowest \( P_e \). For the multi-hop relaying case, the optimization problem is stated as follows

\[ \begin{align*}
A_{\text{S}}^{\text{mep}}, A_{\text{F_k}}^{\text{mep}}, W_d^{\text{mep}} &= \arg\min_{A_{\text{S}}, A_{\text{F_k}}, W_d} P_e(A_{\text{S}}, A_{\text{F_k}}, W_d) \\
\text{s.t.} & \quad \text{(1)} \quad \text{Tr}(A_F \left(\sigma_0^2 C_r^{-1} H_{sr} A_S A_S^H (C_r^H)^{-1} + I_{N_r}\right) A_F^H) \leq P_{r,k}, \\
 & \quad \text{(2)} \quad \sigma_0^2 \text{Tr}(A_S^H A_S) \leq P_t, \quad \text{for} \; k = 1, 2, \ldots, K.
\end{align*} \]  \hfill (21)

We have opted for the projected steepest descent (PSD) [9] for solving our constrained optimization problem, because it was found beneficial in [8]. The initial condition for all of them are chosen to be the LMMSE solution except for the cognitive case, where an MC based initial solution is chosen. This is because unless the matrices involved are strongly rank-deficient and hence non-invertible, it is reasonable to assume that the MEP solution will be in this neighborhood [8]. For the case of multi-hop relaying, even the simplest LMMSE solution has no closed-form expression. Hence, in that case, we opted for using a random initial condition for the LMMSE case and invoked the LMMSE solution for the MEP based one.
\[ P_{\text{e},i} = \frac{1}{2} \left| Q \left( \frac{\mathbb{R} \left( (w_j)^H H x \right) \mathbb{R} (x_j)}{\sqrt{\frac{1}{2} (w_j)^H C w_i}} \right) \right| \left( \frac{\mathbb{R} \left( (w_j)^H H x \right) \mathbb{R} (x_j)}{\sqrt{\frac{1}{2} (w_j)^H C w_i}} \right) + Q \left( \frac{\mathbb{R} \left( (w_j)^H H x \right) \mathbb{R} (x_j)}{\sqrt{\frac{1}{2} (w_j)^H C w_i}} \right) \right| \]

\[ \approx \frac{K}{L} \sum_{j=1}^{L} \exp \left( -\frac{m \sigma^2_j}{2} \right) + \frac{1}{L} \sum_{j=1}^{L} \exp \left( -\frac{m \sigma^2_j}{2} \right) \] where \( a_{1,j} = \frac{\mathbb{R} \left( (w_j)^H H x \right) \mathbb{R} (x_j)}{\sqrt{\frac{1}{2} (w_j)^H C w_i}} \) and \( a_{2,j} = \frac{\mathbb{R} \left( (w_j)^H H x \right) \mathbb{R} (x_j)}{\sqrt{\frac{1}{2} (w_j)^H C w_i}} \).

(14)

Let us now study the BER performance of the proposed method against LMMSE_MC methods for all the above-mentioned MIMO-relay configurations. We consider a non-dispersive Rayleigh fading i.i.d channel with unit variance for each complex element of the channel matrix of the various links. We have used perfect channel for our simulation. The RN’s SNR is defined as \( \text{SNR}_1 = 10 \log_{10} \left( \frac{P_r}{\sigma^2} \right) \) dB, where \( \sigma^2 \) is the power of each \( x_i \), which is set to \( \frac{P_r}{N_d} \). The DN’s SNR is defined as \( \text{SNR}_2 = 10 \log_{10} \left( \frac{P_r}{N_d \sigma^2} \right) \) dB. The SNR1 is kept at 20.5 dB. \( I/P \sigma^2 = 1 \) dB. Our simulation results are averaged over 1000 channel realizations per SNR value. We summarize the simulation parameters in Table III.

IV. NUMERICAL RESULTS

Let us now study the BER performance of the proposed method against LMMSE_MC methods for all the above-mentioned MIMO-relay configurations. We consider a non-dispersive Rayleigh fading i.i.d channel with unit variance for each complex element of the channel matrix of the various links. We have used perfect channel for our simulation. The RN’s SNR is defined as \( \text{SNR}_1 = 10 \log_{10} \left( \frac{P_r}{\sigma^2} \right) \) dB, where \( \sigma^2 \) is the power of each \( x_i \), which is set to \( \frac{P_r}{N_d} \). The DN’s SNR is defined as \( \text{SNR}_2 = 10 \log_{10} \left( \frac{P_r}{N_d \sigma^2} \right) \) dB. The SNR1 is kept at 20.5 dB. \( I/P \sigma^2 = 1 \) dB. Our simulation results are averaged over 1000 channel realizations per SNR value. We summarize the simulation parameters in Table III.

In this work, we have designed only the SN-RN-DN link of the various configurations.
BER performance is poorer for over a flat Rayleigh fading channel without the channel estimation. Fig. 5a (SNR = 5 dB) that the MEP method achieves a BER of $10^{-2}$ at the SNR of $\approx 14.2$ dB, whereas its MC counterpart achieves the same BER at the SNR of $\approx 16.7$ dB. Hence, the MEP based relay design attains an overall SNR gain of about 2.5 dB at the BER of $10^{-2}$. This gain is further increased for higher SNRs. As expected, the BER performance is poorer for $P_t = 0$ dBm, as observed in Fig. 5b. Fig. 6 shows a capacity comparison. We observe that the capacity of the MEP method is poorer as expected.

**Parallel relay:** This solution relies on finding the best link from the set of parallel relay links using $K = 4$. For each link, we have kept the total relay power at 5 dBm. It can be observed in Fig. 7a that the MEP method attains the BER of $10^{-3}$ at the SNR of about 10.2 dB, whereas its LMMSE counterpart achieves the same BER at the SNR of $\approx 13$ dB. Hence, the MEP based relay design attains an overall SNR gain of about $\approx 2.8$ dB at the BER of $10^{-3}$.

**Multi-hop relay:** Let us now embark on characterizing a multi-hop MIMO relay link with 16-QAM constellation. We opted for $N_r = 2$ for all the intermediate RNs. We have chosen $K = 2$, i.e., two serial relay links. For each link, we have kept the total relay power at 5 dBm. It can be observed in Fig. 7b that the MEP method attains the BER of $2\times10^{-2}$ at the SNR of about 17.5 dB, whereas its LMMSE counterpart achieves the same BER at the SNR of $\approx 20$ dB. Hence, the MEP based relay design attains an overall SNR gain of almost 2.6 dB at the BER of $2\times10^{-2}$.

![Fig. 4. A typical complexity comparison between the LMMSE and MEP methods for multi-hop relay design, varying the $N_d$ only.](image)

![Fig. 5. BER vs. SNR performance of the SRD link design for a cognitive MIMO-relay based on the MEP method along with the MC method [5] over a flat Rayleigh fading channel without the channel estimation. $N_f, N_r, N_d, N_p = 2$, $P_t$ is constrained to 5 dBm as shown in Table-III](image)

![Fig. 6. Shows a capacity comparison.](image)

### Table III

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_f, N_r, N_d, N_p$</td>
<td>2</td>
</tr>
<tr>
<td>$P_t$ (Each relay link)</td>
<td>5 dBm</td>
</tr>
<tr>
<td>Constellation</td>
<td>QPSK, QAM</td>
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<tr>
<td>$SNR_1$ (Each Relay link)</td>
<td>20, 5 dB</td>
</tr>
<tr>
<td>$K$</td>
<td>4(Parallel), 2(Multi-hop)</td>
</tr>
</tbody>
</table>

![Complexity vs. SNR](image)
In this correspondence, we have extended the MEP based framework to the design of various types of relaying configurations. We have considered cognitive, parallel and multi-hop relaying. Cost functions have been developed and optimization frameworks have been conceived. Numerical simulations have shown considerable BER performance improvements in all these cases.

**References**


