We know that Huffman codes achieve $L^P(X)$. Let us build the Huffman tree for the given pmf to calculate this quantity (Figure 1).

![Huffman tree](image)

Figure 1: Huffman tree for the pmf $\left(\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{4}\right)$. Codeword assignment shown in red.

It is easy to calculate the average length of this code to be

$$L^P(X) = 1 \cdot \frac{3}{8} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8}$$

$$= 2.$$

Now, we need to calculate the average length of Shannon-Fano-Elias code. Call it $L_{SFE}(X)$.

$$L_{SFE}(X) = \frac{1}{8} \cdot \left\lceil -\log \frac{1}{8}\right\rceil + 1 + \frac{1}{4} \cdot \left\lceil -\log \frac{1}{4}\right\rceil + 1 + \frac{3}{8} \cdot \left\lceil -\log \frac{3}{8}\right\rceil + 1 + \frac{1}{4} \cdot \left\lceil -\log \frac{1}{4}\right\rceil + 1$$

$$= \frac{1}{8} \cdot 4 + \frac{1}{4} \cdot 3 + \frac{3}{8} \cdot 3 + \frac{1}{4} \cdot 3$$

$$= 3.125.$$

Thus, the exact redundancy of the Shannon-Fano-Elias code for this pmf is $3.125 - 2 = 1.125$.

Note that, in case of Shannon-Fano-Elias code, to calculate the average codeword length, unlike Huffman code, we did not need to build the code itself.