This paper studies the effect of coding on the energy consumption of multihop communications in Wireless Sensor Networks. In these networks, there are strict energy consumption constraints because the battery capacity at each node is limited and the goal is to maximize the lifetime of the network. An analytical radio model for energy consumption is used to study the effect of different codes on energy consumption. Tradeoffs between coding overhead and energy consumption per information bit for different channel bit error probabilities and different numbers of hops are computed. For a multihop scenario the numerical results show the tradeoffs between the codes’ energy efficiency, the channel conditions, and the number of hops.

I. INTRODUCTION
Recent developments in wireless communication techniques and in micro-electronics have made possible the construction of low-cost Wireless Sensor Networks (WSNs). In recent years numerous research studies have been carried out in the area of sensor networking to evaluate low-cost and low-power WSNs. With current technology, the wireless communication between sensor nodes is the greatest energy consumer and thus one effective way to decrease the energy consumption in the node’s transceiver is to optimize the energy efficiency of this communication link. The effects of coding and multihop topology on energy consumption have been studied separately [1], [2]. This paper describes a follow up study on how the use of coding affects the energy consumption of multihop communications in WSNs. In the calculations a slow Rayleigh fading channel is assumed, to take into account the different channel conditions encountered across different links in a multihop scenario.

II. RADIO MODEL
Transmission and reception tasks are the largest consumers of energy in wireless embedded networks. A radio model such as the one presented in [1] is often used to represent and analyze this energy consumption. Fig. 1 illustrates the energy $E_L(m,d)$ spent when sending $m$ bits over a wireless hop link of distance $d$.

$$E_L(m,d) = E_T(m,d) + E_R(m) \tag{1}$$

where $E_T$ is the energy spent at the transmitter and $E_R$ the energy spent at the receiver. The transmitter energy consumption can be expressed as:

$$E_T(m,d) = E_{TC}(m) + E_{TA}(m,d) \tag{2}$$

$E_{TC}$ is the energy consumed by the transmitter circuitry and $E_{TA}$ is the energy required by the transmitter amplifier to achieve an acceptable signal-to-noise ratio at the receiver. Assuming a linear relationship for the energy spent per bit by the transmitter and receiver circuitry, Equation 2 can be further simplified to:

$$E_T(m,d) = m(e_{TC} + me_{TA}d^\alpha) \tag{3}$$

$$E_R(k) = me_{RC} \tag{4}$$

where $e_{TC}$, $e_{RC}$ are hardware dependent constants. An explicit expression for $e_{TA}$ can be derived as [2]:

$$e_{TA} = \frac{\left(\frac{S}{N}\right)^{\alpha}}{\frac{(NF_{Rx})(N_0)(BW)}{(G_{ant})(\eta_{amp})(R_{bit})}} \tag{5}$$

where $(S/N)$ is the desired signal-to-noise ratio at the receiver’s demodulator, $NF_{Rx}$ is the receiver noise figure, $N_0$ is the thermal noise floor in a 1 Hz bandwidth, $BW$ is the channel noise bandwidth, $\lambda$ is the wavelength in meters, $\alpha$ is the path loss exponent, $G_{ant}$ is the antenna gain, $\eta_{amp}$ is the transmitter power efficiency and $R_{bit}$ is the raw bit rate in bits per second. This expression for $e_{TA}$ can be used for those cases where a particular hardware configuration is being considered. The dependence of $e_{TA}$ on the $(S/N)$ can be made more explicit if Equation 5 is rewritten as:
\[
e_{\text{TA}} \approx \zeta \left( \frac{S}{N} \right)
\]
\[
\zeta = \frac{(NF_{\text{Rx}})(N_0)(BW)}{(G_{\text{ant}})(\eta_{\text{amp}})(R_{\text{bit}})} \alpha \]

Equation 6 highlights the relationship between \( e_{\text{TA}} \) and the probability of bit error \( p \), which depends on \( (S/N) \). In this paper a variable transmission power scenario is assumed, which means that the radio can dynamically adjust its transmission power so a desired \( (S/N) \), at the receiver is guaranteed.

III. MULTIHOP ENERGY CONSUMPTION

Let us consider a linear sensor array as shown in Fig. 2, which has also been used in other studies [3]:

![Figure 2. Linear sensor array model.](image)

For link \( i \) the probability of bit error is denoted as \( p_i \). The data packet length is \( m \) bits. For the analysis below assume that a Forward Error Correction (FEC) mechanism is being used. \( p_{\text{link}} (i) \) denotes the probability of receiving a packet with uncorrectable errors. The conventional use of FEC is that at each hop the packet is accepted and delivered to the next stage, which in this case means being forwarded to the next node downstream. Assuming a variable transmission power mode the energy consumed in sending a packet from the \( n_{\text{th}} \) node to the sink (using a multihop routing that uses the downstream neighbor as a relay node) can be computed as:

\[
E_{\text{linear}} = m \left[ e_{\text{TC}} + e_{\text{TA}} (d_i) \right] + \sum_{i=2}^{n} \left[ e_{\text{TC}} + e_{\text{RC}} + e_{\text{TA}} (d_i) (\alpha) \right]
\]

(7)

The characteristic distance \( d_{\text{char}} \) is defined as:

\[
d_{\text{char}} = \sqrt{\frac{e_{\text{TA}} + e_{\text{RC}}}{e_{\text{TA}} (\alpha - 1)}}
\]

(11)

where \( \alpha \) is the path loss exponent of the channel, which is typically between 2 and 4. In this paper \( \alpha = 3 \) is used. The characteristic distance can be interpreted as the optimal distance of a single hop for which the energy consumption is minimized. Note that \( d_{\text{char}} \) depends on the hardware radio parameters \( e_{\text{TC}}, e_{\text{RC}} \) and through \( e_{\text{TA}} \) it also depends on \( (S/N) \).

IV. WIRELESS CHANNEL MODEL

The channel characteristics affect the reliability of any communication link. The bit error rate is a useful metric when comparing the energy consumption of various communication methods. A common modulation technique used in WSNs is a non-coherent (envelope or square-law) detector with binary orthogonal FSK signals. For this case the probability of bit error for a non-fading channel is [4]:

\[
p(\gamma_b) = \frac{1}{2} e^{-\frac{\gamma_b}{2}}
\]

(12)

For typical multihop sensor networks it is more reasonable to assume a Rayleigh slow fading channel attenuation. In this case,

\[
p_{\text{FSK}} = \frac{1}{2 + \gamma_b}
\]

(13)

where \( \gamma_b \) is the average signal-to-noise ratio.

V. LINEAR CODES

Common codes are used in this study so that the trade-offs between coding overhead and energy consumption per information bit can be easily illustrated. Short linear block codes of the form \( (m, k, d_{\text{min}}) \) have been used, where \( m \) is the length of the code word, \( k \) is the number of information bits and \( d_{\text{min}} \) is the minimum distance of the code. Table 1 shows the codes’ parameters, where \( t \) is the error correction capability.

<table>
<thead>
<tr>
<th>Code</th>
<th>( m )</th>
<th>( k )</th>
<th>( d_{\text{min}} )</th>
<th>Rate</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>0.57</td>
<td>1</td>
</tr>
<tr>
<td>Golay</td>
<td>23</td>
<td>12</td>
<td>7</td>
<td>0.52</td>
<td>3</td>
</tr>
<tr>
<td>Shortened Hamming</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>0.50</td>
<td>1</td>
</tr>
<tr>
<td>Extended Golay</td>
<td>24</td>
<td>12</td>
<td>8</td>
<td>0.50</td>
<td>3</td>
</tr>
</tbody>
</table>

TABLE 1. Code parameters.
Long BCH codes have also been used. Table 2 shows these codes’ parameters.

<table>
<thead>
<tr>
<th>Code</th>
<th>(m)</th>
<th>(k)</th>
<th>(t)</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCH</td>
<td>511</td>
<td>304</td>
<td>25</td>
<td>0.59</td>
</tr>
<tr>
<td>BCH</td>
<td>511</td>
<td>277</td>
<td>28</td>
<td>0.52</td>
</tr>
<tr>
<td>BCH</td>
<td>511</td>
<td>157</td>
<td>51</td>
<td>0.31</td>
</tr>
</tbody>
</table>

The probabilities of errors in the code words for the different codes depend on the channel’s bit error rate and the properties of the codes. Since a variable transmission power mode is being assumed, the probability of the bit error for each link is fixed and thus \(P_c = (1 - P_{\text{link}})^n\), where \(n\) is the number of hops. The value of \(P_{\text{link}}\) depends on the received signal-to-noise ratio as well as on the modulation and coding method. For FSK-modulation with non-coherent detection and assuming ideal interleaving the probability of a code word being in error is approximated by [4]:

\[
P_c(\varepsilon) = \sum_{h=1}^{\infty} \binom{m}{h} \varepsilon^h (1-\varepsilon)^{m-h}
\]  

(14)

where \(\varepsilon\) represents the bit error probability, \(P_{\text{FSK}}\), of the channel.

The packet error probability can be then calculated as:

\[
P_e = 1 - P_c = 1 - (1 - P_{\text{link}})^n
\]  

(15)

where \(P_{\text{link}}\) is \(P_c(\varepsilon)\).

The probability of successful transmission of a packet is

\[
P_{\text{success}} = (1 - P_e)
\]  

(16)

The expected number of transmissions for successful end-to-end packet transmission can be calculated as:

\[
E_{\text{end-to-end-Tx}} = \frac{1}{P_{\text{success}}}
\]  

(17)

The expected energy consumption per information bit is then:

\[
E_{i-bit} = E_{\text{end-to-end-Tx}} \left( \frac{E_{\text{linear}}}{k} \right)
\]  

(18)

Equation 18 can be used to compare how different codes affect the energy consumption in a linear multihop scenario.

VI. RESULTS

The following numerical analysis has been carried out to study the effects of coding on the energy consumption of a multihop WSN scenario. For these calculations, the size \(D\) of the linear array is assumed to be 1000 meters. The radio parameter values used are shown in Table 3. The energy consumption per information bit for different codes and with different numbers of hops was calculated using the equations from the previous sections.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{\text{Rx}})</td>
<td>10dB</td>
</tr>
<tr>
<td>(N_0)</td>
<td>-173.8 dBm/Hz or 4.17 * 10^{-21} J</td>
</tr>
<tr>
<td>(R_{\text{bit}})</td>
<td>115.2 Kbits</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0.3 m</td>
</tr>
<tr>
<td>(G_{\text{ant}})</td>
<td>-10dB or 0.1</td>
</tr>
<tr>
<td>(G_{\text{amp}})</td>
<td>0.2</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>3</td>
</tr>
<tr>
<td>(BW)</td>
<td>For FSK-modulation, it can be assumed to be the same as (R_{\text{bit}})</td>
</tr>
<tr>
<td>(\varepsilon_{TC})</td>
<td>50nJ/bit</td>
</tr>
<tr>
<td>(\varepsilon_{RC})</td>
<td>50nJ/bit</td>
</tr>
</tbody>
</table>

Fig. 3 shows \(d_{\text{char}}\) as a function of the bit error probability, \(p_{\text{FSK}}\), for the radio parameters listed in Table 3. Values of \(p_{\text{FSK}}\) were chosen between 3*10^{-2} and 10^{-4}. It can be seen that \(d_{\text{char}}\) increases when the bit error probability increases. To have low values for the bit error probability the transmission power must be high which results in a short characteristic distance. Low transmission power values increase the bit error probability as well as the characteristic distance.

![Characteristic distance as a function of the bit error probability](image-url)

Figure 3. Characteristic distance as a function of the bit error probability \(p_{\text{FSK}}\).

Fig. 4 shows \(E_{i-bit}\) for different short linear codes when the number of nodes is 70. In this case the distance of one hop is 14.3 meters, which is close to the optimal distance between two neighbor nodes when the bit error probability of the channel is about 10^{-2}. The simplest (7, 4, 3) Hamming code appears to be the most energy efficient for low bit error probability (BEP) values but the differences between the various codes are very small. The Golay codes start to be the most energy efficient when the BEP value is over 0.01. It is apparent that the error correction properties of the stronger codes become more useful
as $p_{FSK}$ increases. Fig. 5 gives a view of the energy consumption for very low BEP values and includes the case when no coding is used.

Fig. 4. Expected energy consumption, $n=70$.  

Fig. 5. Expected energy consumption, $n=70$.  

Fig. 6 shows the energy consumption for the same radio parameters but with $n=10$. In this case the single hop distance (100 m) is larger than the characteristic distance corresponding to the range of BEP values shown and not only the expected energy consumption values are larger than the ones shown in Fig. 4, but also coding does not have a major impact on such consumption.

Fig. 7 shows the energy consumption when $n = 300$. This value of $n$ corresponds to the distance of one hop to be near the optimal value for $p_{FSK} = 10^{-4}$. The energy efficiency of the various codes has increased for low values of BEP when compared to the results shown in Fig. 4. Fig. 7 also shows that error correction is useful when the number of hops is high. When the number of hops is high the energy efficiency of the simpler codes starts to decrease as $p_{FSK}$ increases.

Fig. 7. Expected energy consumption per information bit, $n=300$.  

Fig. 8 shows the expected energy consumption for the long BCH codes used in this study and when the number of hops is 70. In this case the energy efficiency of the simpler codes is better for a wide range of BEP values. The stronger error correcting codes become more useful only when the BEP value is large.

Fig. 8. Expected energy consumption per information bit for BCH codes, $n=70$.  

As was the case with the shorter codes, Fig. 9 shows that when the number of hops is 300, the long stronger codes are more useful when the BEP values are large. Stronger codes prevent excessive error probability accumulation as the number of hops increases.

Fig. 10 shows the energy efficiency of the $(7, 4, 3)$ code for different values of $n$. These results show that using 300 hops is the most energy efficient method when the BEP value is very low. On the other hand using 70 hops is more energy efficient when the BEP value is greater than $3 \times 10^{-3}$. Obviously, the energy efficiencies of the different
scenarios depend on the actual BEP value. The energy consumption for all the cases starts to increase when the BEP value is over $10^{-2}$ because the $(7, 4, 3)$ code is not good enough for high BEP values as shown in Figs. 4 and 7. It is observed that the lowest energy consumption is achieved when the number of hops is 70 and the BEP value is between $3 \times 10^{-3}$ and $2.5 \times 10^{-2}$.

III. CONCLUSIONS

This paper studies how different channel codes affect the energy consumption in multihop Wireless Sensor Networks. Energy efficiencies of the codes were calculated using an analytical radio model with different values for the number of hops. The results show that depending on the channel conditions, the efficiencies of the codes vary. When the channel conditions are good, there is not much difference between the codes. As expected, error control becomes more useful as the channel conditions get worse. For a particular BEP value the lowest energy consumption is achieved when the single hop distance matches the transceiver characteristic distance. The node density is application dependent. If the application is such that the number of hops is larger than the optimum value for the transceiver, then the use of error control coding is recommended to contain the accumulation of probability error with each hop. The model used assumes that the transmitter can dynamically adjust its transmission power so that the desired signal-to-noise ratio is guaranteed at the receiver. This is an unrealistic assumption because a set of fixed transmission power levels is normally used. Further work is needed to study fixed transmission power scenarios and also to find efficient coding techniques for practical applications of WSNs.

References


