Performance Analysis of a Delay Tolerant Network by State Aggregation

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Abstract—We consider a delay tolerant network, with mobile nodes modelled as performing independent random walks on a graph. We compute the mean time to convey a message from a mobile source to a fixed destination on the graph, via mobile relay nodes adopting a two-hop relay protocol. The complexity of computing the mean delay is high in a large network. Therefore, we propose two low complexity methods to approximately compute the mean delay via state aggregation. Similar methods can be used to obtain the performance of other routing protocols in such a network.

Keywords—Delay tolerant networks, vehicular networks, delay analysis, low complexity methods, graph partitioning, state aggregation.

I. INTRODUCTION

A delay tolerant network (DTN) lacks continuous network connectivity. Such a network should tolerate large delays due to intermittent connectivity of nodes. Disruption in connectivity may occur because of the limits of wireless radio range, sparsity of mobile nodes, energy resources, attack, and noise. This is an emerging area of research and there is progressive growth of activity as it is useful in many situations of interest ([1], [2]).

Researchers have worked to achieve interoperability between the nodes of a DTN by proposing new architectures ([3]). In [4], the authors suggest the use of node mobility to significantly increase the throughput capacity of a delay tolerant network. Various mobility models have been used to address the connectivity deficit in vehicular delay tolerant networks ([5]-[9]). Further, researchers have proposed novel protocols to solve the connectivity and large delay issues in a DTN. There is also considerable work on performance analysis of DTNs operating with suitable routing protocols ([10]-[15]). In [11], the authors analyse performance for random waypoint and random direction mobility models for epidemic routing, two-hop routing and also a newly introduced variant of two-hop relay protocol. They provide closed form expressions and asymptotic results when the number of nodes is large for packet delivery delay and the energy needed. In [12], the authors use Queueing Petri Nets to accurately model complex DTNs. The generated bundles (data) have delivery delay that depends on the size of the bundle and the link bandwidth. They find a closed form expression for the expected delivery delay of a bundle in a generalized model by deriving the underlying reachability graph and computing the hitting time of the associated Semi-Markov process. In [13], the authors analyse the throughput in a very generalised framework for any mobility model in stationarity. Relays are used to increase the throughput, i.e., there are no additional copies of messages. They also use state aggregation to reduce complexity, to preserve the sum of stationary probabilities of the elements of the aggregated state.

In our work we do not assume stationarity, and relays are used to reduce the delay by replicating the messages. We approximate the distributions of important components of the end to end delay, performing state aggregation and reducing complexity to estimate the mean delivery delay.

The rest of the paper is organized as follows. In Section II, we describe the model. In Section III, we calculate exactly the mean time for a packet to be delivered. In Section IV, we propose two low computational complexity methods to approximately calculate the same metric. In Section V, we compare the three methods and present our results. Section VI concludes the paper.

II. SYSTEM MODEL

We consider a network of roads (links) in a geographic area which can be represented by a planar connected graph, for example as in Figure 1. There are $N$ mobile vehicles moving independently on these roads. At every intersection the choice of the next road for a mobile is made at random with predefined probabilities for each outbound road ([8], [9]). The time spent by a mobile on each road is assumed to be exponentially distributed with a parameter depending on the road. This can be represented as a simple random walk on the graph. The nodes may generate messages at random times which need to be transmitted to a destination node which is stationary. The mobile nodes can transmit messages on a wireless channel. For every link we define its parallel link set as the set of links between which communication is possible i.e., when two (or more) mobile nodes belong to the same parallel link set, we obtain a line of sight (LoS) wireless channel. Then these nodes can successfully receive messages from one another; otherwise they cannot, because then there may be shadowing due to buildings, trees, etc. The notion of parallel link set is a generalisation of the LoS channel that can be obtained between the directional links representing a bidirectional road. Further, a mobile node can directly transmit to the destination node if it is on the road on which the destination node is located or on a road that belongs to the parallel link set of this road. We assume that the time taken to transmit a packet is negligible.

We tag a packet generated at $t = 0$ on mobile $k$ (say $k=1$, without loss of generality) and calculate the mean time taken to deliver this packet to the destination. The packet can be delivered by using any relay routing protocol. When the tagged packet (or its copy) is given to a relay (a mobile node), an additional copy of the tagged packet is generated and the relay with the new copy takes an independent path to reach the destination. The packet is said to be delivered when a copy of the tagged packet reaches the destination.

We denote the set of all unidirectional links of the geographical area by $L$ and its cardinality by $n_L$. Let $F_i(t) = 1$ if the tagged packet is available at mobile $i$ at time $t$; otherwise it is 0. Also $F_i(t) = 1$, for all $t \geq 0$. We use $G_i$ to...
define flag changes: \(G_i(t) = F_i(t^+)\). We use \(L_i(t)\) and \(M_i(t)\) to represent the link that mobile \(i\) is on at time \(t\) and \(t^+\) respectively. \(L_i(t)\) and \(M_i(t)\) belong to \(LL, X(l)\) is a subset of \(L\) that contains all links that lead out of the terminating intersection of link \(l\). We denote the parallel link set of link \(l\) by \(DL(l)\), a subset of \(L\). We denote the probability of transition of mobile node \(i\) from link \(l\) to link \(m\) at transition instant \(T_i\), \(P_i(l_i(T_i) = m|L_i(T_i) = l)\) by \(P_i(m|l)\). We assume that the transition probability is time stationary. We denote the holding time of mobile node \(i\) on link \(l\) by \(T_i(l)\). We assume that the holding times are independent and exponentially distributed. We define the system state \(X(t)\) at time \(t\) as \(\{L_i(t), \ldots, X_N(l), F_2(t), \ldots, F_N(t)\}\). We denote the time required to deliver the packet to the destination starting from the system state \(s\) by \(t_s\).

Our objective is to calculate the mean delivery delay, which is defined as the average time taken for a tagged packet generated at time \(t = 0\) to be delivered to the destination node.

### III. Exact Formulation

We consider the case of two-hop relay routing for \(N\) mobiles. In two-hop relay routing, if the source node has not reached the destination, it transmits the message to all the mobile node (relays) it meets, so that a copy is available with the relay for delivery to the destination. A relay node is allowed to copy the message only to the destination.

Due to the memoryless property of the exponentially distributed holding times, \(X(t)\) is a Markov chain. To define state transition probabilities, we first need to identify the possibility of state transition from a system state \((l_1, \ldots, l_N, f_2, \ldots, f_N)\) to \((m_1, \ldots, m_N, g_2, \ldots, g_N)\). We denote vector \(l = (l_1, l_2, \ldots, l_N)\) in terms of coordinates \(i\) and \(-i\) (all the coordinates of the vector other than coordinate \(i\), \(\{1, \ldots, i-1, i+1, \ldots, N\}\)). If mobile \(i\) makes the link transition, \(m_j = l_j\), for all \(j \neq i\). For this \(i\) and vectors \(l, f, m, g\), we define condition \(C_i\) such that \(C_i\) is TRUE, when there is a transition from state \((l, f)\) to state \((m, l_{-i}), g\), else \(C_i\) is FALSE. Let us consider the flag changes that can occur due to the transition of mobile \(i \neq 1\) : (A) \(g_i = f_i\) when the parallel link set containing \(m_i\) does not have mobile 1. (B) \(g_i = 1\) when mobile 1 is located in the parallel link set. Similarly, the flag changes that occur when mobile 1 makes the link transition are \(g_i = 1\) when mobile \(j \neq 1\) is located on a link of the parallel link set that contains \(m_1\). These flag change conditions are condensed as follows: \(C_i\) is

\[
(m_j = l_j) \text{ AND } \left( (f_j = g_j) \text{ AND } (l_j \notin DL(m_1)) \right) \text{ OR } \left( (g_j = 1) \text{ AND } (l_j \notin DL(m_1)) \right) \text{ for all } j \neq 1,
\]

and \(C_i, i \neq 1\) is

\[
(m_j = l_j) \text{ AND } \left( (f_i = g_i) \text{ AND } (m_i \notin DL(l_1)) \right) \text{ OR } \left( (g_i = 1) \text{ AND } (m_i \notin DL(l_1)) \right) \text{ for all } j \neq i.
\]

The state transition probability for \(X(t)\),

\[
P \left( m_1, \ldots, m_N, g_2, \ldots, g_N | l_1, \ldots, l_N, f_2, \ldots, f_N \right) = P_i \left( m_i | l_i \right) P^{(i)}(t_i) = \min \{ T_i^{(j)} \}, \text{ if } C_i \text{ is true,}
\]

\[
0, \text{ else.}
\]

The mean delivery delay for this case can be written recursively as

\[
E[\tau_{fi}] = \left\{ \begin{array}{ll}
0, & \text{if } l_1 \in S \text{ or } l_i \in S, f_i = 1, \\
\sum_{m} \sum_{m_i} E[\tau_{(m_i,l_{-i}),g}] P \left( (m_i, l_{-i}), g | l, f \right) + E[\min \{ T_i^{(j)} \}], & \text{else}.
\end{array} \right.
\]

(2)

where, \(f\) is the vector of flags for all mobiles before the transition, while \(g^{(i)}\) is the vector of flags for all mobiles just after the transition of mobile \(i\) and is defined as follows,

\[
g_j^{(i)} = f_j, j \neq i \text{ and } g_i^{(i)} = \max(f_j, 1_{(m_i \in DL(l_j))}), \text{ for all } i \neq 1, \text{ since } F_1(t) = 1 \text{ for all } t \geq 0. \text{ The above equations are the backward induction (or value iteration) equations for solving for the conditional mean } E[\tau_{fi}].
\]

These recursive equations can be solved to yield the mean delivery delay for all initial states. A similar set of equations can be written and solved for any connected graph and any number of mobile nodes.

We now show existence and uniqueness of solution to the above problem. We denote the matrix of coefficients of unknowns in equation set (2) by \(A\). The system of equations can be written as \(v = Av + b\). We note that \(A\) is sub-stochastic. The maximum eigenvalue of \(A\) is less than one \([16]\) and hence, the matrix \((I-A)\) is invertible. The solution to the problem, \((I-A)^{-1}b\), exists and is unique.

For the single mobile case, when there are multiple destinations \(\{S_1, 1 \leq i \leq N_d\}\), and the mobile starts from link \(l\), we can calculate \(P^{sink}(i)\), the probability of reaching destination \(S_i\) before reaching any \(S_j\), \(j \neq i\), as follows. For all \(i, 1 \leq i \leq N_d\), we have

\[
P^{sink}(i) = \left\{ \begin{array}{ll}
1_{(l \in S_i)}, & \text{if } l \in S, \\
\sum_{m} P^{sink}(i)P \left( m | l \right), & \text{else.}
\end{array} \right.
\]

This is a useful parameter required in the next section.

### IV. Approximate Formulation

#### A. Motivation

The state space for \(N\) mobile nodes on a \(n_L\) (unidirectional) link graph has states \(\{l_1, l_2, \ldots, l_N, f_2, \ldots, f_N\}\) with the total number of states \(2n_LN^2/2\). For large geographical areas, the value of \(n_L\) is large, and even for a moderate value of \(N\) the number of states is very large. Thus, computing the mean time for the packet to reach the destination via equations (2) can be very complex. We propose low computational complexity methods to compute the mean time.

#### B. Brief description

We edge contract the nodes to make a smaller graph. If an edge \((i, j)\) is contracted then node \(i, j\) become one node and all the edges on \(i\) and \(j\) connect to this new node. This way we aggregate the nodes in a geographical area (zone), which are well connected to each other, into one node, while one zone has few connections to another zone. Methods for such a graph clustering exist and are well researched ([19], [20]). An example of this is shown in Figure 1 and 2. We assume that the time to leave a zone from any interior link is an exponentially distributed random variable with parameter depending on the link and the zone. The motivation for this assumption and the
mean for the exponential distribution will be provided below. We call the newly formed graph a super-graph. We assume that the super-graph obtained is a simple graph, i.e., there can be at most one link between any pair of zones. The edges that join any two zones are called super-links. Super-link $SL_i(t)$ represents the zone of the mobile $i$ at time $t$ and the entry link through which it entered the zone. The zone that contains the destination node is called the sink zone, and the zones that contain the mobiles at $t = 0$ are called the initial zones. If the super graph obtained after graph partitioning is not a simple graph, i.e., the number of roads between a pair of zones is more than one, then we can construct an equivalent simple graph. This construction involves estimating the probability distribution of the entry link last used by a mobile, when both its current and previous zones are known. Estimating this probability distribution is similar to the steps involved in obtaining equation (7) shown later.

C. State aggregation and super-graph representation

The exponential assumption above is motivated from Theorem 1.4(a) in [18], which states that

$$\sup_i|P_{\pi_{1,stat}}(T_A > t) - \exp(-t/E_{\pi_{1,stat}}T_A)| \leq \Delta, \quad (4)$$

where $T_A$ is the first passage time to visit set $A$, $\Delta = C\tau/E_{\pi_{1,stat}}T_A\{1 + \log^+(E_{\pi_{1,stat}}T_A/\tau)\} < 1$, $C$ is a numerical constant, not dependent on the Markov chain $X_t$, and $\tau$ is defined by

$$\tau = \min\{t : ||P_t(X_t \in \cdot) - \pi_{stat}|| \leq 1/(2e)\}$$

for all states $i$. The exponential assumption above is motivated from Theorem 1.4(a) in [18].

Here $||\alpha|| = \frac{1}{2} \sum |\alpha_i|$ is the total variation norm. Informally, if $E_{\pi_{1,stat}}T_A > \tau$, then $\Delta$ is small and hence $T_A$ is almost exponential.

For graph representation, the super-links could be used to represent the position of a mobile node. Let us consider the two mobile case with a single destination $l_3$ located in zone 3. The initial positions of mobile 1 and mobile 2 are $l_1$ (in zone 1) and $l_2$ (in zone 2) respectively. The first sojourn time of mobile 1 in zone 1 depends on $l_1$ and it is not same as the sojourn time at any other revisit to zone 1. Hence, the same super-link cannot represent the first sojourn and remaining sojourns in a zone. So, we include auxiliary initial zones with the same neighbouring zones as the initial zone. The auxiliary initial zones cannot be revisited. The sole purpose of these auxiliary initial zones is to represent the initial state with parameters possibly different from other states. Similarly, an auxiliary sink zone is included and this zone can be reached only through the original sink zone. The delivery delay from the auxiliary sink zone is 0. An example of this is shown in Figure 2. We have created an auxiliary sink zone $3^*$ and two auxiliary initial zones for the first sojourn out of the respective initial zones. The inclusion of the auxiliary zones helps simplify the properties required to well define the geographical area with a super-graph.

D. New state space description

Consider the new state space description $Y(t) = \{SL_1(t), \cdots, SL_N(t), F_2(t), \cdots, F_N(t)\}$. The loss of information due to state space reduction has to be compensated by calculating and estimating a few parameters. As suggested earlier, the time spent by a mobile on a super-link between zone $i$ and $j$ is the time spent in zone $j$ when entered from zone $i$ and it is exponentially distributed. The mean of this parameter can be easily found from the equation set (2) by setting $N=1$, initial link to be the entry link of zone $j$ from zone $i$, and the set of exit links of zone $j$ as the destination. The transition probability of a single mobile from a super-link to another can be similarly obtained from equation set (3). A relatively difficult question is that of the flag transition, since two mobiles can be located in the same zone and may or may not be able to communicate with each other. But, given the initial location of the two mobiles in the zone and the flag state, we can modify equation set (3) to calculate $P_{\text{copy}}(t)$, the probability of the availability of the tagged packet at the relay by the time of the first exit from the zone of any of the two mobiles.

$$P_{\text{copy}}^{\text{exit}}_{l_1,l_2,f} = \begin{cases} f, & \text{if } l_1 \text{ or } l_2 \in \text{ExitLinks}(j), \\ \sum_{m_1,m_2,g} P_{\text{copy}}(m_1,m_2,g_{l_1,l_2,f}), & \text{else}. \end{cases} \quad (5)$$

Now, consider the case when a mobile enters a zone which already contains another mobile, the location of the mobile entering the zone is exactly known if the previous zone is known, but the exact location of the mobile already inside is not known (due to state aggregation). If the distribution Dist-Inside of the mobile already inside, is given, then we can use it to find the expected probability of packet availability at relay, among other useful parameters. The two methods proposed below differ chiefly in how this distribution is estimated.

E. Method I : using PASTA

We define an entry and exit for an extended zone by using distinct hysteresis thresholds, i.e., geographically, the boundary that separates the inside and outside of a zone is different for an entry event and an exit event. This is further explained in Figure 3. $T_1$ and $T_2$ are thresholds for outgoing mobiles, and $T_3$ and $T_4$ are thresholds for incoming mobiles. This defines the graph partitioning for various parameters to be calculated. We assume that the time taken for a mobile to revisit a zone is i.i.d and exponentially distributed. This is a reasonable
assumption, since a large number of short revisit sojourns as shown in Figure 4, that contribute to the quick decay rate of the tail distribution, are avoided by using distinct hysteresis thresholds in the extended zone model. In these figures, event $T_1$, $l$ occurs when a mobile crosses threshold $T_2$ from right to left. Further, Figure 5 is shown for an example in Figure 6 with bidirectional links and with mean time spent on any link being 5.0. The process defined by the number of revisit events is now a Poisson process. For a zone, the state process of a (the first) mobile in this zone (while the mobile has not left the zone) is a Markov chain which eventually attains stationarity. By PASTA, at the event of the second mobile revisiting this zone, the process above is found in stationarity. Dist-Inside, the distribution of the first mobile at the arrival event, is its stationary distribution in the zone.

\[ P^\text{inside} = \begin{cases} 1_{\{l_1 \in \text{zone}\}}, & \text{if } l_2 \in \text{EntryLinks}, \\ 0, & \text{if } l_1 \text{ is outside the zone}, \\ \sum_{m_1, m_2} P^\text{inside}(m_1, m_2|l_1, l_2), & \text{else} \end{cases} \]

(7)

The mean delivery delay for the two mobile case from state $(s_{l_1}, s_{l_2}, f)$ for both the approximate methods can be written recursively as

\[ E[\tau_{s_{l_1}, s_{l_2}, f}] = \begin{cases} 0, & \text{if } s_{l_1} \in S, \\ 0, & \text{if } s_{l_2} \in S, \ f = 1, \text{ and else} \\ \sum_{m_1} E[\tau_{s_{m_1}, s_{l_2}, g^{(1)}|s_{l_1}, s_{l_2}, f}], & \text{if } s_{l_1} \in S, \ s_{l_2} \in S, \ f = 1, \text{ and else} \end{cases} \]

\[ + \sum_{m_2} E[\tau_{s_{l_1}, s_{m_2}, g^{(2)}|s_{l_1}, s_{l_2}, f}], \]

where, $S$ is the set of auxiliary sink nodes. $T^{(i)}_{s_{l_1}, s_{l_2}, f}$ is the delivery delay from state $(s_{l_1}, s_{l_2}, f)$ and $Pr(s_{m_1}, s_{m_2}, g|s_{l_1}, s_{l_2}, f)$ is the state transition probability.

**V. RESULTS**

We test our methods on geographies represented by graphs in Figures 1 and 6. The average time taken to traverse each of the links is specified by the link mean vector, which is generated randomly for each environment. In an environment,
we select a set of initial states and sink states. We then find the error between the methods in calculating the time taken to reach the sink state for each element of a set of initial states. The different environments we consider are as follows:

1) Graph in Figure 6 with the links between vertices 1 and 2 as sink states and link mean vector generated independently with pmf $[0.3 \ 0.4 \ 0.3]$ on elements [7 10 13].

2) Graph in Figure 6 with the links between vertices 1 and 2 as sink states and link mean vector generated independently with pmf $[0.3 \ 0.4 \ 0.3]$ on elements [5 10 15].

3) Graph in Figure 6 with the links between vertices 1 and 2 as sink states and link mean vector generated independently with pmf $[0.3 \ 0.4 \ 0.3]$ on elements [5 10 15].

In Table I, the average delivery delay found through exact method is denoted by $T_i$, and this parameter estimated through Method I and Method II are denoted by $T_1$ and $T_2$ respectively. We define error between exact method and approximate Method $i$ as $E_{error} = (T_i - T)/T_i$, for $i = 1, 2$. The set of detailed results shown in Table I are calculated for initial states which are close to each other. The results show that the error function changes slowly in a neighbourhood and hence we use Monte Carlo integration to estimate the average error. For each of the considered environments, the initial states were chosen randomly from the uniform distribution and the average error results are shown in Table II. Method I has fewer parameters to be calculated and is based on theory in [17]. The approximation of the distribution of revisit time in combination with the approximation of holding time in a zone is not very good.

VI. CONCLUSION

We conclude that the approximate methods suggested give reasonably accurate results with far lower memory requirements and hence lower complexity for the examples considered. Method II performs better than Method I as it is based on better approximations. The suggested methods perform very well when the zones represent gated communities, which have few outgoing links and can be contracted into single zones without excessive loss of detail.

REFERENCES


